King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP144A Nuclear Physics

Summer 2005

Time Allowed: THREE Hours

Candidates should answer all SIX parts of SECTION A, and no more than TWO questions from SECTION B. No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Physical Constants

Permittivity of free space	$\epsilon_0 = 8.854 \times 10^{-12}$ F m^{-1}
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}
Speed of light in free space	$c = 2.998 \times 10^{8}$ m s^{-1}
Gravitational constant	$G = 6.673 \times 10^{-11}$ N m^2 kg^{-1}
Elementary charge	$e = 1.602 \times 10^{-19}$ C
Electron rest mass	$m_\mathrm{e} = 9.109 \times 10^{-31}$ kg
Unified atomic mass unit	$m_\mathrm{u} = 1.661 \times 10^{-27}$ kg = 931.5 MeV c^{-2}
Proton rest mass	$m_\mathrm{p} = 1.673 \times 10^{-27}$ kg
Neutron rest mass	$m_\mathrm{n} = 1.675 \times 10^{-27}$ kg
Planck constant	$h = 6.626 \times 10^{-34}$ J s
Boltzmann constant	$k_\mathrm{B} = 1.381 \times 10^{-23}$ J K^{-1}
Stefan-Boltzmann constant	$\sigma = 5.670 \times 10^{-8}$ W m^{-2} K^{-4}
Gas constant	$R = 8.314$ J mol^{-1} K^{-1}
Avogadro constant	$N_\mathrm{A} = 6.022 \times 10^{23}$ mol^{-1}
Molar volume of ideal gas at STP	$= 2.241 \times 10^{-2}$ m^3
One standard atmosphere	$P_0 = 1.013 \times 10^{5}$ N m^{-2}
One electron volt	1 eV $= 1.602 \times 10^{-19}$ J
Coulomb constant	$e^2/4\pi\epsilon_0 = 1.440$ MeV fm

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SECTION A - Answer all SIX parts of this section

1.1) Assume that the neutron has the same mass as the proton, and that atomic nuclei are spherical with an average radius $\overline{R} = (1.2 \times 10^{-15})A^{1/3}$ m, where A is the mass number. Show that, to a first approximation, which you should state, the nuclear mass density is independent of A, and determine its value.

[7 marks]

1.2) The half life of ${}^{132}_{53}\mathrm{I}$ is 2.3 hours. A sample containing ${}^{132}_{53}\mathrm{I}$ has a measured activity of 10^5 Bq. How many atoms of ${}^{132}_{53}\mathrm{I}$ are contained in the sample?

[7 marks]

1.3) On the basis of the quark properties in Table 1.3, suggest the possible quark content of protons and neutrons. Show that your suggestions are consistent with relevant properties of these nucleons.

[7 marks]

Quark	Charge	Strangeness	Baryon Number
u	+2e/3	0	1/3
d	−e/3	0	1/3
s	−e/3	−1	1/3

TABLE 1.3

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1.4) Natural Uranium consists of two isotopes in the ratio $^{235}U/^{238}U = 7.3 \times 10^{-3}$. The mean life of ^{235}U is 1.03×10^9 years, and that of ^{238}U is 6.49×10^9 years. Assume that these isotopes existed in equal amounts at the time the Earth was formed. From this calculate the age of the Earth.

[7 marks]

1.5) The following reaction is observed to occur in a head-on collision between an electron and a positron with equal kinetic energies

$$e^+ + e^- \rightarrow \pi^+ + \pi^-$$

Given that the rest mass of the $\pi^\pm$ mesons is 139.56 MeV c^{-2} calculate the kinetic energy of the electrons at which this reaction can just occur.

[7 marks]

1.6) Using the information given in Table 1.6 below, determine whether the following reactions can occur through the strong interaction:

(a) $\pi^0 + n \rightarrow K^+ + \Sigma^-$

(b) $\pi^- + p \rightarrow \pi^- + \Sigma^+$

(c) $\pi^- + p \rightarrow \Sigma^- + \Sigma^+$

Particle	Baryon Number	Strangeness
π^-	0	0
π^0	0	0
K^+	0	+1
Σ^-	+1	−1
Σ^+	+1	−1
n	+1	0
p	+1	0

TABLE 1.6

[7 marks]

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SECTION B - Answer TWO questions

2 (a) An atomic nucleus with an odd value of mass number A has a binding energy B in MeV that is given by the semi-empirical expression

$$B(Z,A) = 15.84A - 18.33A^{2/3} - 0.71\frac{Z(Z-1)}{A^{1/3}} - 23.2\frac{(A-2Z)^2}{A}$$

where Z is the atomic number.

Explain the origin of the terms in the above equation. Describe the additional term that must be added if the expression is to be used to describe the binding energy of nuclei with even A values.

[20 marks]

(b) Use the semi-empirical binding energy expression given in (a) to find the atomic number of the most stable element having $A = 209$. Hence determine the number of neutrons in the nucleus, and comment briefly on your result.

[10 marks]

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3 A non-relativistic particle of mass m and charge ze undergoes Rutherford scattering from a heavy nucleus of charge Ze. The recoil of the heavy nucleus can be ignored. You may assume that all nuclei are spherical and have an average radius $\overline{R} = 1.2. \times A^{1/3}$ fm, where A is the mass number, and that their charge acts as if concentrated at the centre of the sphere.

(a) Sketch the trajectory of the non-relativistic particle. Mark on your diagram the impact parameter b and the symmetry axis of the particle's trajectory, and explain briefly the significance of these terms.

[8 marks]

(b) At any time t the location of the scattered particle is given relative to the nucleus by the coordinates (r, ϕ), where the angle ϕ is measured relative to the symmetry axis of the scattering. Show that the impact parameter b is given by:

$$b = (r^2 \frac{d\phi}{dt})/v_0$$

where v_0 is the initial velocity of the particle.

[5 marks]

(c) Write down an expression for the electrostatic Coulomb force $\vec{F}$ acting on the particle, in terms of charges and the distance r from the nucleus.

[2 marks]

(d) Show that the distance $d_{\min}$ of closest approach of the particle to the nucleus for head-on collisions is given by:

$$d_{\min} = Zze^2/2\pi\epsilon_0 m v_0^2.$$

[7 marks]

(e) An α particle of kinetic energy 2 MeV is fired from a large distance at a heavy $^{197}_{79}$Au nucleus in a head-on collision. Assuming that the nucleus remains at rest throughout the scattering, calculate the closest distance of approach, giving your answer in fm (1 fm $=10^{-15}$ m).

[3 marks]

(f) Estimate the minimum kinetic energy that an α particle must have if it is to be fired from a large distance and just come into contact with a $^{197}_{79}$Au nucleus.

[5 marks]

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4 (a) Name the three basic ways in which nuclei decay. Which of these is related to the quantum-mechanical phenomenon of *tunnelling*? Explain this phenomenon qualitatively, using sketches where appropriate.

[6 marks]

(b) An unstable nucleus which is initially at rest emits an α particle. Show that the kinetic energy K of the recoiling daughter nucleus is:

$$K = \frac{M_\alpha}{M} K_\alpha$$

where M is the mass of the daughter nucleus, M_α is the mass of the α-particle, and K_α is the kinetic energy of the α particle.

[5 marks]

(c) Show that the amount Q of energy released in the α-decay of (b) is related to K_α by:

$$Q = K_\alpha \left(1 + \frac{M_\alpha}{M} \right) \ .$$

[5 marks]

(d) Calculate Q in MeV for the decay :

$$^{226}_{88}\text{Ra} \rightarrow ^{222}_{86}\text{Rn} + ^{4}_{2}\text{He} \ .$$

where the atomic masses are: $M_{^4_2\text{He}} = 4.002603\ m_\text{u}$, $M_{^{222}_{86}\text{Rn}} = 222.017574\ m_\text{u}$ and $M_{^{226}_{88}\text{Ra}} = 226.025406\ m_\text{u}$.

[7 marks]

(e) Determine the kinetic energy K_α of the α-particle in MeV for the reaction in part (d), and thus show that the recoil energy of the daughter nucleus is only a small percentage of K_α.

[7 marks]

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