King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP144A Nuclear Physics

Summer 2004

Time Allowed: THREE Hours

Candidates should answer no more than SIX parts of SECTION A, and no more than TWO questions from SECTION B. No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Physical Constants

Permittivity of free space	$\epsilon_0 = 8.854 \times 10^{-12}$ F m^{-1}
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}
Speed of light in free space	$c = 2.998 \times 10^{8}$ m s^{-1}
Gravitational constant	$G_{\rm N} = 6.673 \times 10^{-11}$ N m^2 kg^{-1}
Elementary charge	$e = 1.602 \times 10^{-19}$ C
Electron rest mass	$m_{\rm e} = 9.109 \times 10^{-31}$ kg
Unified atomic mass unit	$m_{\rm u} = 1.661 \times 10^{-27}$ kg
Proton rest mass	$m_{\rm p} = 1.673 \times 10^{-27}$ kg
Neutron rest mass	$m_{\rm n} = 1.675 \times 10^{-27}$ kg
Planck constant	$h = 6.626 \times 10^{-34}$ J s
Boltzmann constant	$k_{\rm B} = 1.381 \times 10^{-23}$ J K^{-1}
Stefan-Boltzmann constant	$\sigma = 5.670 \times 10^{-8}$ W m^{-2} K^{-4}
Gas constant	$R = 8.314$ J mol^{-1} K^{-1}
Avogadro constant	$N_{\rm A} = 6.022 \times 10^{23}$ mol^{-1}
Molar volume of ideal gas at STP	$= 2.241 \times 10^{-2}$ m^3
One standard atmosphere	$P_0 = 1.013 \times 10^{5}$ N m^{-2}
One year in seconds	1 year $= 3.16 \times 10^{7}$ s
Conversion factor for $m_{\rm u}$	1 $m_{\rm u} = 931.5$ MeV c^{-2}

SEE NEXT PAGE

SECTION A - Answer SIX parts of this section

1.1) Give the definition of the unified atomic mass unit, and from this, by using the Avogadro constant, show that the value of the unified atomic mass unit is 1.661×10^{-27} kg.

[7 marks]

1.2) Most nuclei are spherical, to a very good approximation, with average radius $\overline{R} = 1.2 \times A^{1/3} \times 10^{-15}$ m, where A is the atomic mass number. Assuming that protons and neutrons have approximately the same mass, find an approximate expression for the mass of a nucleus of mass number A. State the main reason for the approximate nature of this formula. Use the formula to show that the nuclear mass density is independent of A, and determine its value.

[7 marks]

1.3) Determine which of the following interactions are possible. Justify your answers briefly.

(a) ${}^{A}_{Z}X^{N} \rightarrow {}^{A}_{Z+1}Y^{N-1} + e^{+} + \bar{\nu}_{e}$

(b) ${}^{A}_{Z}X^{N} \rightarrow {}^{A}_{Z-1}Y^{N+1} + e^{+} + \nu_{e}$

(c) $\mu^{-} \rightarrow e^{-} + \bar{\nu}_{e}$

[7 marks]

1.4) Describe qualitatively the basic steps of an induced fission reaction involving ^{235}U. Sketch the nuclear shape at various stages of the fission process in the context of the liquid drop model of the nucleus. Use stability arguments to justify your answer.

[7 marks]

1.5) Name the three basic ways in which nuclei decay. Which of these is related to the quantum-mechanical phenomenon of *tunneling*? Explain this phenomenon qualitatively, using sketches where appropriate.

[7 marks]

SEE NEXT PAGE

1.6) Consider the table

Particle	**B**	**S**
π^-	**0**	**0**
π^0	**0**	**0**
K^+	**0**	**+ 1**
Σ^-	**+1**	**−1**
Σ^+	**+ 1**	**−1**
n	**+1**	**0**
p	**+1**	**0**

TABLE 1

where the symbols have their usual meanings. Explain whether each of the reactions (a) and (b) could proceed via strong interactions.

(a) $\pi^0 + n \rightarrow K^+ + \Sigma^-$

(b) $\pi^- + p \rightarrow \pi^- + \Sigma^+$

[7 marks]

1.7) What is meant (a) by a lepton and (b) a baryon. What particles exist *always* in the final stage of a decay series of a baryon? To which of the categories (a), (b) do neutrinos belong?

[7 marks]

SEE NEXT PAGE

1.8) The Ω-baryon is a strange particle, with strangeness quantum number $S = -3$. From this data and Table 2 determine the quark composition of Ω and its electric charge. The symbols in the table have their usual meanings.

Quark	Q(e)	S	C	B'	T'
u	+2/3	0	0	0	0
d	−1/3	0	0	0	0
s	−1/3	−1	0	0	0
c	+ 2/3	0	+1	0	0
b	−1/3	0	0	−1	0
t	+2/3	0	0	0	+1

TABLE 2

[7 marks]

SEE NEXT PAGE

SECTION B - Answer TWO questions

2. Consider the β-decay of ^{14}C:

$$^{14}_{6}\mathrm{C} \rightarrow ^{14}_{7}\mathrm{N} + e^{-} + \bar{\nu}_e$$

which is used for radioactive dating. The half life of ^{14}C is 5730 years. Assume that the ratio of the concentrations of the isotopes of ^{14}C and ^{12}C in the molecules of CO_2 in the atmosphere is constant, and equal to 1.3×10^{-12}.

(a) Explain briefly the principle of radioactive dating.

[5 marks]

(b) A piece of wood of mass 25 g was discovered in an excavation. The sample has ^{14}C activity of 250 decays/min.

(i) Determine the decay constant of ^{14}C.

[3 marks]

(ii) Calculate the number of ^{12}C and ^{14}C atoms in the sample when it was cut.

[7 marks]

(iii) Determine the initial ^{14}C activity.

[2 marks]

(iv) Hence calculate how many years have elapsed since the sample was cut.

[13 marks]

SEE NEXT PAGE

3. The α-decay of an unstable nucleus which is initially at rest releases energy Q.

(a) Show that the kinetic energy K of the recoiling daughter nucleus is:

$$K = \frac{M_\alpha}{M} K_\alpha$$

where M is the mass of the daughter nucleus, M_α is the mass of the α-particle, and K_α is the kinetic energy of the α particle.

[7 marks]

(b) Show that Q and K_α are related by:

$$Q = K_\alpha \left(1 + \frac{M_\alpha}{M}\right) \ .$$

[8 marks]

(c) Calculate Q in MeV for the decay :

$$^{226}_{88}\mathrm{Ra} \rightarrow ^{222}_{86}\mathrm{Rn} + ^{4}_{2}\mathrm{He} \ .$$

where the atomic masses are: $M_{^4_2\mathrm{He}} = 4.002603\ m_\mathrm{u}$, $M_{^{222}_{86}\mathrm{Rn}} = 222.017574\ m_\mathrm{u}$ and $M_{^{226}_{88}\mathrm{Ra}} = 226.025406\ m_\mathrm{u}$.

[8 marks]

(d) Determine (in MeV) the kinetic energy K_α of the α-particle for the reaction in part (c), and thus show that the recoil energy of the daughter nucleus is only a small percentage of K_α.

[7 marks]

4. **(a)** Sketch the trajectory of a non-relativistic particle of charge ze, which undergoes Rutherford scattering from a heavy nucleus of charge Ze. Ignore recoil of the heavy nucleus throughout. Define in your diagram the impact parameter b and the symmetry axis of the particle's trajectory.

[8 marks]

(b) At any time t the location of the scattering particle is given relative to the nucleus by the coordinates (r, ϕ), where the angle ϕ is measured relative to the symmetry axis of the scattering. Express the impact parameter b in terms of $\mathrm{d}\phi/\mathrm{d}t$.

[8 marks]

(c) Write down an expression for the electrostatic Coulomb force $\vec{F}$ acting on the particle, in terms of the distance r from the nucleus and the charges.

[2 marks]

(d) Determine the closest distance d_{min} of the particle from the nucleus, for head-on collisions, in terms of the initial velocity v_0 of the particle, its mass m and the charges of the particle and the nucleus.

[7 marks]

(e) Determine the closest distance in fm (1 fm $=10^{-15}$ m) for a head-on non-relativistic Rutherford scattering of an α-particle from a heavy nucleus $^{197}_{79}\mathrm{Au}$, if the initial kinetic energy of the α particle is 10 MeV. Assume that the nucleus remains at rest throughout the scattering.

Assume for the purposes of this exercise that all nuclei are spherical, with an average radius $\overline{R} = 1.2. \times A^{1/3}$ fm, where A is the mass number, and that their charge is concentrated at the centre of the sphere. Then determine the minimum kinetic energy of the incident α particle such that the two nuclei come into contact.

[The Coulomb constant $e^2/4\pi\epsilon_0 = 1.440$ MeV fm]

[5 marks]

SEE NEXT PAGE

5. **(a)** The semi-empirical mass formula for the atomic mass m of a nucleus ${}^{A}_{Z}\mathrm{X}^{N}$ reads:

$$m = (A - Z)m_{\mathrm{n}} + Z(m_{\mathrm{p}} + m_{\mathrm{e}}) -$$
$$\frac{a_V A}{c^2} + \frac{a_S A^{2/3}}{c^2} + \frac{a_C Z(Z-1)}{A^{1/3}c^2} + \frac{a_{\mathrm{sym}}(A-2Z)^2}{Ac^2} + \frac{a_P}{A^{3/4}c^2}$$

Discuss the physical meaning of each of the terms in this formula, explaining their dependence on the mass number A.

[8 marks]

(b) For odd A, show that the value of Z which corresponds to the most stable isobar ${}^{A}\mathrm{X}$ is given by the integer value closest to:

$$Z_{\mathrm{min}} = \frac{(m_{\mathrm{n}} - m_{\mathrm{p}} - m_{\mathrm{e}})c^2 + a_C A^{-1/3} + 4a_{\mathrm{sym}}}{2a_C A^{-1/3} + 8a_{\mathrm{sym}} A^{-1}}$$

[8 marks]

Sketch the graph m vs. Z for fixed odd A nuclides.

[6 marks]

(c) Determine the decay processes of nuclei with (i) $Z > Z_{\mathrm{min}}$ and (ii) $Z < Z_{\mathrm{min}}$.

[8 marks]

FINAL PAGE