

King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CP144A Nuclear Physics

Summer 2004

Time Allowed: THREE Hours

Candidates should answer no more than **SIX** parts of **SECTION A**, and no more than **TWO** questions from **SECTION B**. No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

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Physical Constants

Permittivity of free space	$\epsilon_0 = 8.854 \times 10^{-12} \text{ F m}^{-1}$
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$
Speed of light in free space	$c = 2.998 \times 10^8 \text{ m s}^{-1}$
Gravitational constant	$G_N = 6.673 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-1}$
Elementary charge	$e = 1.602 \times 10^{-19} \text{ C}$
Electron rest mass	$m_e = 9.109 \times 10^{-31} \text{ kg}$
Unified atomic mass unit	$m_u = 1.661 \times 10^{-27} \text{ kg}$
Proton rest mass	$m_p = 1.673 \times 10^{-27} \text{ kg}$
Neutron rest mass	$m_n = 1.675 \times 10^{-27} \text{ kg}$
Planck constant	$h = 6.626 \times 10^{-34} \text{ J s}$
Boltzmann constant	$k_B = 1.381 \times 10^{-23} \text{ J K}^{-1}$
Stefan-Boltzmann constant	$\sigma = 5.670 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Gas constant	$R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$
Avogadro constant	$N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$
Molar volume of ideal gas at STP	$= 2.241 \times 10^{-2} \text{ m}^3$
One standard atmosphere	$P_0 = 1.013 \times 10^5 \text{ N m}^{-2}$
One year in seconds	$1 \text{ year} = 3.16 \times 10^7 \text{ s}$
Conversion factor for m_u	$1 m_u = 931.5 \text{ MeV c}^{-2}$

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SECTION A - Answer SIX parts of this section

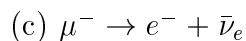
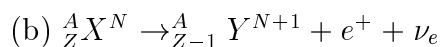
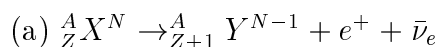
- 1.1) Give the definition of the unified atomic mass unit, and from this, by using the Avogadro constant, show that the value of the unified atomic mass unit is 1.661×10^{-27} kg.

[7 marks]

- 1.2) Most nuclei are spherical, to a very good approximation, with average radius $\bar{R} = 1.2 \times A^{1/3} \times 10^{-15}$ m, where A is the atomic mass number. Assuming that protons and neutrons have approximately the same mass, find an approximate expression for the mass of a nucleus of mass number A . State the main reason for the approximate nature of this formula. Use the formula to show that the nuclear mass density is independent of A , and determine its value.

[7 marks]

- 1.3) Determine which of the following interactions are possible. Justify your answers briefly.



[7 marks]

- 1.4) Describe qualitatively the basic steps of an induced fission reaction involving ${}^{235}\text{U}$. Sketch the nuclear shape at various stages of the fission process in the context of the liquid drop model of the nucleus. Use stability arguments to justify your answer.

[7 marks]

- 1.5) Name the three basic ways in which nuclei decay. Which of these is related to the quantum-mechanical phenomenon of *tunneling*? Explain this phenomenon qualitatively, using sketches where appropriate.

[7 marks]

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1.6) Consider the table

Particle	B	S
π^-	0	0
π^0	0	0
K^+	0	+1
Σ^-	+1	-1
Σ^+	+1	-1
n	+1	0
p	+1	0

TABLE 1

where the symbols have their usual meanings. Explain whether each of the reactions (a) and (b) could proceed via strong interactions.

(a) $\pi^0 + n \rightarrow K^+ + \Sigma^-$

(b) $\pi^- + p \rightarrow \pi^- + \Sigma^+$

[7 marks]

1.7) What is meant (a) by a lepton and (b) a baryon. What particles exist *always* in the final stage of a decay series of a baryon? To which of the categories (a), (b) do neutrinos belong?

[7 marks]

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- 1.8) The Ω -baryon is a strange particle, with strangeness quantum number $S = -3$. From this data and Table 2 determine the quark composition of Ω and its electric charge. The symbols in the table have their usual meanings.

Quark	Q(e)	S	C	B'	T'
u	+2/3	0	0	0	0
d	-1/3	0	0	0	0
s	-1/3	-1	0	0	0
c	+ 2/3	0	+1	0	0
b	-1/3	0	0	-1	0
t	+2/3	0	0	0	+1

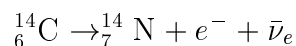
TABLE 2

[7 marks]

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SECTION B - Answer TWO questions

2. Consider the β -decay of ^{14}C :



which is used for radioactive dating. The half life of ^{14}C is 5730 years. Assume that the ratio of the concentrations of the isotopes of ^{14}C and ^{12}C in the molecules of CO_2 in the atmosphere is constant, and equal to 1.3×10^{-12} .

- (a) Explain briefly the principle of radioactive dating.

[5 marks]

(b) A piece of wood of mass 25 g was discovered in an excavation. The sample has ^{14}C activity of 250 decays/min.

- (i) Determine the decay constant of ^{14}C .

[3 marks]

(ii) Calculate the number of ^{12}C and ^{14}C atoms in the sample when it was cut.

[7 marks]

(iii) Determine the initial ^{14}C activity.

[2 marks]

(iv) Hence calculate how many years have elapsed since the sample was cut.

[13 marks]

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3. The α -decay of an unstable nucleus which is initially at rest releases energy Q .

(a) Show that the kinetic energy K of the recoiling daughter nucleus is:

$$K = \frac{M_\alpha}{M} K_\alpha$$

where M is the mass of the daughter nucleus, M_α is the mass of the α -particle, and K_α is the kinetic energy of the α particle.

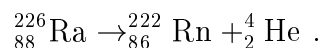
[7 marks]

(b) Show that Q and K_α are related by:

$$Q = K_\alpha \left(1 + \frac{M_\alpha}{M} \right) .$$

[8 marks]

(c) Calculate Q in MeV for the decay :



where the atomic masses are: $M_{{}_2^4\text{He}} = 4.002603 m_u$, $M_{{}_{86}^{222}\text{Rn}} = 222.017574 m_u$ and $M_{{}_{88}^{226}\text{Ra}} = 226.025406 m_u$.

[8 marks]

(d) Determine (in MeV) the kinetic energy K_α of the α -particle for the reaction in part (c), and thus show that the recoil energy of the daughter nucleus is only a small percentage of K_α .

[7 marks]

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4. (a) Sketch the trajectory of a non-relativistic particle of charge ze , which undergoes Rutherford scattering from a heavy nucleus of charge Ze . Ignore recoil of the heavy nucleus throughout. Define in your diagram the impact parameter b and the symmetry axis of the particle's trajectory.

[8 marks]

- (b) At any time t the location of the scattering particle is given relative to the nucleus by the coordinates (r, ϕ) , where the angle ϕ is measured relative to the symmetry axis of the scattering. Express the impact parameter b in terms of $d\phi/dt$.

[8 marks]

- (c) Write down an expression for the electrostatic Coulomb force \vec{F} acting on the particle, in terms of the distance r from the nucleus and the charges.

[2 marks]

- (d) Determine the closest distance d_{\min} of the particle from the nucleus, for head-on collisions, in terms of the initial velocity v_0 of the particle, its mass m and the charges of the particle and the nucleus.

[7 marks]

- (e) Determine the closest distance in fm ($1 \text{ fm} = 10^{-15} \text{ m}$) for a head-on non-relativistic Rutherford scattering of an α -particle from a heavy nucleus ${}_{79}^{197}\text{Au}$, if the initial kinetic energy of the α particle is 10 MeV. Assume that the nucleus remains at rest throughout the scattering.

Assume for the purposes of this exercise that all nuclei are spherical, with an average radius $\bar{R} = 1.2 \times A^{1/3} \text{ fm}$, where A is the mass number, and that their charge is concentrated at the centre of the sphere. Then determine the minimum kinetic energy of the incident α particle such that the two nuclei come into contact.

[The Coulomb constant $e^2/4\pi\epsilon_0 = 1.440 \text{ MeV fm}$]

[5 marks]

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5. (a) The semi-empirical mass formula for the atomic mass m of a nucleus ${}^A_ZX^N$ reads:

$$m = (A - Z)m_n + Z(m_p + m_e) - \frac{a_V A}{c^2} + \frac{a_S A^{2/3}}{c^2} + \frac{a_C Z(Z - 1)}{A^{1/3} c^2} + \frac{a_{\text{sym}}(A - 2Z)^2}{Ac^2} + \frac{a_P}{A^{3/4} c^2}$$

Discuss the physical meaning of each of the terms in this formula, explaining their dependence on the mass number A .

[8 marks]

- (b) For odd A , show that the value of Z which corresponds to the most stable isobar ${}^A X$ is given by the integer value closest to:

$$Z_{\text{min}} = \frac{(m_n - m_p - m_e)c^2 + a_C A^{-1/3} + 4a_{\text{sym}}}{2a_C A^{-1/3} + 8a_{\text{sym}} A^{-1}}$$

[8 marks]

Sketch the graph m vs. Z for fixed odd A nuclides.

[6 marks]

- (c) Determine the decay processes of nuclei with (i) $Z > Z_{\text{min}}$ and (ii) $Z < Z_{\text{min}}$.

[8 marks]

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