King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CP/1400 Classical Mechanics and Special Relativity

Summer 1999

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A,
and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

Separate answer books must be used for each Section of the paper.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Section A - Answer **SIX** parts of this section

1.1) State Kepler's second law of planetary motion. Use this law to show that the speeds v_a and v_p of a planet at aphelion and perihelion respectively are related by the equation $\frac{v_a}{v_p} = \frac{r_p}{r_a}$, where r_a and r_p are the distances of the planet from the sun at the two positions.

[7 marks]

1.2) One end of a light inextensible cord of length l is tied to a fixed point O, while the other end is attached to a small sphere of mass m. The sphere is launched to move in a vertical circular path, with O as its centre. If the tension in the cord is zero when the sphere is at its maximum height, show that the speed of the sphere at this point is $\sqrt{gl}$. Determine the speed of the sphere when it reaches its lowest point. (Neglect friction.)

[7 marks]

1.3) What is meant by an *impulse*? State how impulse and momentum are related. A player strikes a stationary golf ball (mass 46 g) which leaves the head of the club with a velocity of 200 km h^{-1} at an angle of 33° to the ground. The club head is in contact with the ball for 1.5 ms. Determine (a) the impulse on the ball; (b) the impulse on the club; (c) the average force exerted on the ball.

[7 marks]

1.4) Show that, under appropriate conditions, which should be specified, a simple pendulum executes simple harmonic motion.

[7 marks]

1.5) What is meant by a totally inelastic collision? During an ice-dance routine, a skater moving in a straight line with a velocity of 25 km h^{-1} picks up his stationary partner, whose mass is 75% of his own. What is the velocity of the pair immediately after the manoeuvre if they continue along the initial linear path?

[7 marks]

1.6) The centre-to-centre separation of ions in the diatomic molecule hydrogen chloride is 0.13 nm, while the masses of the hydrogen and chlorine atoms are, respectively, 1.67×10^{-27} kg and 5.80×10^{-26} kg. Determine the position of the centre of mass of the molecule.

[7 marks]

1.7) What fundamental property of light forms a basis of the special theory of relativity? Show that events which occur simultaneously in one reference frame may not do so in another moving relative to the first.

[7 marks]

1.8) Explain the importance of an internationally-agreed system of units in scientific investigations. What are the principles by which the fundamental SI units of mass, length and time are established?

[7 marks]

Section B - Answer **TWO** questions from this section

2) Show, from first principles, that the moment of inertia of a uniform solid cylinder of mass m and radius r about its axis is $\frac{1}{2}mr^2$.

[10 marks]

The cylinder described above starts from rest and rolls, without slipping, down a ramp inclined at an angle θ to the horizontal.

(a) Show that the angular acceleration of the cylinder is $\dfrac{2g\sin\theta}{3r}$;

[5 marks]

(b) determine the linear acceleration of the cylinder;

[5 marks]

(c) what is the frictional force acting between the cylinder and the ramp?

[5 marks]

(d) find the kinetic energy of the cylinder at time t after it starts rolling.

[5 marks]

3) A rocket, of initial mass (including fuel) m_i, is launched from a spacecraft in deep space. The exhaust gases released by the rocket have a speed u relative to its engine.

Show that

(a) the thrust (or propulsive force) experienced by the rocket is Ru, where R is the rate of fuel consumption (mass per unit time);

[7 marks]

(b) when its mass has fallen to m, the speed, v, of the rocket relative to the spacecraft is given by $v = u\ln\left(\dfrac{m_i}{m}\right)$.

[8 marks]

To boost its speed and change its direction, the rocket is set on a course to pass a distant planet and the engine is cut. While the rocket is remote from the planet, the angle between the essentially linear path of the rocket and the path of the planet is θ, while after passing the planet and finally escaping from its gravitational field the trajectories of rocket and planet are parallel. Show that the final speed of the rocket relative to the sun is $v_f = v_p + \sqrt{v_i^2 + v_p^2 - 2v_i v_p\cos\theta}$, where v_i is the approach speed of the rocket relative to the sun while v_p is the speed of the planet relative to the sun. (Neglect the effect of the sun on the motion of the rocket.)

Show that the maximum value of v_f is $2v_p+v_i$, and sketch the trajectory for this situation.

[15 marks]

4) A cube of mass m is given an impulse to propel it across a rough horizontal surface. The cube experiences a resistive force of magnitude bv, where v is the speed and b is a constant. Show that the speed of the cube after the impulse has the form $v = v_0 \exp\left(-\frac{t}{\tau}\right)$.

   Explain the significance of v_0 and τ.

   [10 marks]

   The cube is attached to one end of a horizontal spring of spring constant k, the other end of which is attached to a rigid support. The cube is given a small horizontal displacement from equilibrium along the line of the spring. The displacement, x, of the cube with time, t, can take the form $x = x_0 \exp(-\beta t)\sin(\omega t + \phi)$. Show that $\beta = \frac{b}{2m}$ and $\omega^2 = \frac{k}{m} - \beta^2$.

   [12 marks]

   Sketch graphs to illustrate the different types of motion that are possible, identifying the relationship that holds between the parameters for each type of motion.

   [8 marks]

5) Derive analytical relationships for *time dilation* and *length contraction* which are predicted by the special theory of relativity.

   [16 marks]

   The half-life of muons measured in their rest frame is 2.2 μs, while in the laboratory frame the half-life is found to be 4.1 μs. At what speed are the muons moving relative to the laboratory?

   [7 marks]

   Muons travelling with this speed reach the Earth-based laboratory having passed through 5 km of the Earth's atmosphere. What thickness is the atmosphere as measured by an observer travelling with the muons?

   [7 marks]