King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/1400 Classical Mechanics and Special Relativity

Summer 1997

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A,
and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

Separate answer books must be used for each Section of the paper.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Acceleration due to gravity at the Earth's surface, $g = 9.8$ m s^{-2}
Gravitational constant, $G = 6.7 \times 10^{-11}$ m^3 s^{-2} kg^{-1}
Mass of the Earth = 5.97×10^{24} kg
Radius of the Earth = 6.38×10^{6} m

Section A - Answer **SIX** parts of this section

1.1 Show that the kinetic energy of a rotating body is $I\omega^2/2$, where I is the moment of inertia of the body and ω is its angular velocity.

[7 marks]

1.2 What are meant by (a) *coefficient of restitution* and (b) *conservation of momentum* as used in collision processes. In a snooker game, the cue ball (white) is propelled along the line joining the centres of the balls to collide at a velocity 2 m s^{-1} with a stationary red ball. If the physical characteristics of the two balls are the same, effects of spin are negligible and the collision can be regarded as perfectly elastic, use your answers to (a) and (b) to determine the velocities of the two balls immediately after the collision.

[7 marks]

1.3 Use the principle of energy conservation to show that the escape velocity of a body from the surface of the Earth is about 11.2 km s^{-1}.

[7 marks]

1.4 Write down a differential equation which defines *Simple Harmonic Motion*, and explain carefully the meaning of each term. Show that a motion in which the displacement x is related to time t by the equation $x = a \sin(bt + \varphi)$ is Simple Harmonic, and explain the significance of the constants a, b and φ.

[7 marks]

1.5 Data for the elliptical orbits of the planets Mercury and Jupiter are:

	Mercury	Jupiter
Semi-major axis of orbit (m)	5.79×10^{10}	7.78×10^{11}
Orbital period (yr)	0.24	11.9

Show that these data are in reasonable accord with Kepler's 3rd law of planetary motion.

[7 marks]

1.6 In the Special Theory of Relativity, proper time t_{pr} and improper time t_{im} are related by the equation

$$t_{pr} = t_{im}\sqrt{1-\frac{v^2}{c^2}}\ .$$

Carefully explain the meaning of each symbol in this equation. In the interpretation of experiments to determine the half life of μ-mesons, the half life measured as t_{pr} is found to be 2.2 μs, while that given by t_{im} is 4.0 μs. Deduce the speed of the μ-mesons.

[7 marks]

1.7 Two springs, with spring constants of 3 N m^{-1} and 6 N m^{-1} respectively are joined together in series. Show, with appropriate analysis, that the spring constant of the series combination is 2 N m^{-1}.

[7 marks]

1.8 State Newton's three laws of motion and indicate any limitations on their applicability.

[7 marks]

Section B - Answer **TWO** questions from this section

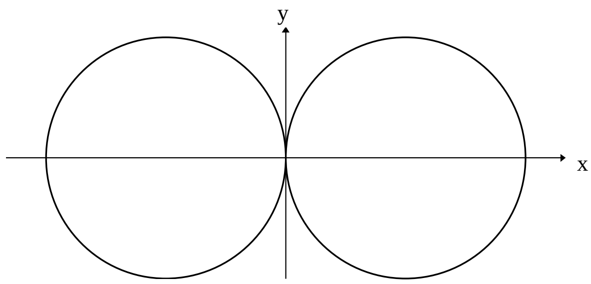
2. Define the *moment of inertia* of a body.

[5 marks]

A thin plate lies in the xy-plane of a Cartesian co-ordinate system. Prove the perpendicular axis theorem for thin plates, which states that $I_z = I_x + I_y$ where I represents moment of inertia and the subscript indicates the axis about which the moment is calculated.

[8 marks]

The moment of inertia of a thin circular plate of mass *M* and radius *r* about an axis through its centre and perpendicular to the plate is $Mr^2/2$. An engine component consists of two thin circular plates joined at their edges and lying in the xy-plane plane to form a figure-of-eight shape (See diagram).The line joining the centres of the plates lies along the *x*-axis and the junction of the plates is at the origin. By using the perpendicular and parallel axis theorems, or otherwise, obtain expressions for the moments of inertia of the component about (a) the *z*-axis and (b) the *y*-axis.



[10 marks]

A torque is applied to the component to cause it to rotate about the *z*-axis. In a separate situation, a torque of the same magnitude is applied to the component to cause it to rotate about the *y*-axis. Demonstrate that the ratio of the angular acceleration produced on the two occasions is 1.2, taking care to indicate which angular acceleration is the greater.

[7 marks]

[Note: The parallel axis theorem states that the moment of inertia (I) of a body about an axis is given by $I = I_c + Mr^2$, where I_c is the moment of inertia of the body about a parallel axis through the centre of mass, M is the mass of the body and r is the separation of the two axes.]

3. A hemisphere of radius 20 cm is fixed with its flat face in contact with a horizontal surface. A small mass is placed on the outside surface of the hemisphere vertically above its centre. The mass is given a small displacement from equilibrium so that it slides, without rolling, over the surface. If frictional effects can be neglected,

(a) draw a 2-D diagram and label the forces acting on the mass while it is in contact with the hemisphere;

[10 marks]

(b) show that it will leave the hemisphere at a position where the radius makes an angle of $\cos^{-1}(2/3)$ with the vertical;

[10 marks]

(c) determine where the mass will strike the horizontal surface.

[10 marks]

4. Draw graphs to illustrate damped, critically damped and overdamped motion, identifying the relationship that holds between the parameters for each type of motion.

[8 marks]

A sphere of mass m is released from rest at time $t = 0$ in a fluid and falls vertically under the influence of gravity. The resistance to motion experienced by the sphere in moving with velocity v through the fluid is $R = bv$ where b is a constant. Write down the equation of motion of the sphere and hence show that its velocity is given by $v = v_0(1 - \exp(-t/t))$. Explain the significance of v_0 and τ.

[10 marks]

The sphere is now suspended in the fluid by a spring (constant = k). The sphere is given a small vertical displacement from equilibrium.

(a) Write down the equation of motion of the system.

(b) Obtain expressions for β and ω in terms of k, b and m for which the equation $x = x_0 \exp(-\beta t)\sin(\omega t + \phi)$ represents the displacement, x, of the sphere as a function of time.

[12 marks]

5. What is the postulate upon which Einstein's Special Theory of Relativity is based? What consequence does this have for the speed of light?

[8 marks]

The Lorentz transformation of co-ordinates between two Cartesian co-ordinate frames of reference (primed and unprimed), the axes of which are initially coincident, with the primed frame moving with velocity v in the x-direction relative to the unprimed frame, is:

$$x' = \gamma(x - vt);\ \ y' = y;\ \ z' = z;\ \ t' = \gamma(t - vx/c^2),$$

where $\gamma = 1/\sqrt{(1 - v^2/c^2)}$.

(a) Show that this transformation reduces to the Galilean transformation when $v << c$.

[6 marks]

(b) Show that velocities transform according to the relationships:

$$v'_x = \frac{v_x - v}{1 - \frac{v v_x}{c^2}};\quad v'_x = \frac{v_y}{\gamma(1 - \frac{v v_x}{c^2})};\quad v'_x = \frac{v_z}{\gamma(1 - \frac{v v_x}{c^2})}.$$

[8 marks]

(c) Use the results obtained in (b) to show that light beams travelling *in vacuo* in the unprimed frame (i) in the x-direction and (ii) in the y-direction have a speed c in the primed frame. Determine their directions in the primed frame.

[8 marks]