Acceleration due to gravity $g = 9.80 \text{ m s}^{-2}$
Speed of light in free space $c = 3.00 \times 10^{8} \text{ m s}^{-1}$
Gravitational constant $G = 6.67 \times 10^{-11} \text{ N m}^{2} \text{ kg}^{-2}$

SECTION A - Answer **SIX** parts of this section

1.1) An amusement park ride consists of a large, hollow cylindrical chamber, of radius 4 m, which rotates about its vertical axis of symmetry. Riders enter the cylinder while it is stationary, and stand with their backs against the chamber wall. When all riders are in position, the chamber is rotated with gradually increasing angular velocity. At some stage during the ride, the floor of the chamber is removed so that it is no longer in contact with the feet of the riders. Draw a clear diagram to indicate the forces which act on one of the riders. Determine the minimum angular velocity of the chamber if the riders are to remain in place on the wall. (The coefficient of friction between a rider and the wall is 0.5.)

[7 marks]

1.2) A uniform spring, with spring constant 5 N m^{-1}, is cut into three pieces of equal length. Determine the spring constant of each piece. The three pieces of spring are mounted in parallel between two rigid bars. Find the equivalent spring constant of the combination.

[7 marks]

1.3) What is meant by the centre of mass of a body? An engine component consists of a metal disc of radius 10 cm from which has been cut a circular hole of radius 5 cm. The centre of the hole lies at a distance of 5 cm from the centre of the disc. Determine the position of the centre of mass of the component.

[7 marks]

1.4) State Newton's second law of motion. A mass moving in a straight line on a rough surface experiences a resistive force proportional to its velocity. Show that the velocity decreases exponentially with time.

[7 marks]

1.5) State Kepler's third law of planetary motion. Mercury and Jupiter have elliptical orbits with semi-major axes of 5.79×10^{10} m and 7.78×10^{11} m respectively. Given that the orbital period of Mercury is 0.24 years, estimate the orbital period of Jupiter.

[7 marks]

1.6) Sketch the world line for a racing cyclist who is on the starting line for 1 min, accelerates from rest to a velocity of 40 km hr^{-1} in 2 min with constant acceleration, moves with this velocity for 2 min before slowing to rest in 2 min with constant deceleration. Calculate the distance travelled by the cyclist.

[7 marks]

SEE NEXT PAGE

1.7) A 5 kg mass on a frictionless horizontal surface is attached to one end of a horizontal spring (spring constant 10 N m^{-1}), the other end of which is clamped. The mass is displaced so that the spring is stretched along its axis beyond its natural length, and then released. Show that the mass will execute simple harmonic motion, and determine its angular frequency. Write down an expression for the displacement of the mass as a function of time, t, given that the mass is released at $t = 0$. (It may be assumed that the spring obeys Hooke's Law during its deformation.)

[7 marks]

1.8) Write down an equation relating force and potential energy. The Mie potential energy for an atom in a diatomic molecule is $U(r) = -\frac{A}{r^m} + \frac{B}{r^n}$, where r is the separation of the atoms and A, B, m and n are positive constants. Show that the equilibrium separation, r_0, of the atoms is given by $r_0^{(m-n)} = \left(\frac{mA}{nB}\right)$.

[7 marks]

SECTION B - Answer **TWO** questions

2) State the principle of conservation of mechanical energy.

[3 marks]

One end of a light *inextensible* cord of length l is attached to a rigid support, while the other end is attached to a small sphere of mass m. The system is set in motion so that the mass moves in a *vertical* circle, with the support at its centre, so that the cord is never slack. If the tension in the cord is zero when the sphere is at its maximum height, show that the speed of the sphere

(a) at its maximum height is $\sqrt{gl}$;

(b) when the cord is horizontal is $\sqrt{3gl}$;

(c) at its lowest point is $\sqrt{5gl}$.

(d) Determine the tension in the cord for each of (b) and (c).

[15 marks]

The sphere is now attached to one end of a light *extensible* cord of natural length l, the other end of which is attached to a rigid support. The sphere is launched to move in a *horizontal* circular path of radius l, in the manner of a conical pendulum. If the speed of the sphere is $\sqrt{gl}$, show that

(e) the angle made by the cord with the horizontal is 45°;

(f) the "spring constant" of the cord is $\frac{\sqrt{2}mg}{(\sqrt{2}-1)l}$;

(g) the energy stored in the cord is $\frac{\sqrt{2}-1}{\sqrt{2}}mgl$.

[12 marks]

SEE NEXT PAGE

3) What are (a) a perfectly elastic collision, (b) a totally inelastic collision and (c) an impulse?

[7 marks]

A stationary block of wood, of mass 0.6 kg, on a horizontal rough surface is struck by a bullet of mass 6 g moving horizontally with a speed of 400 m s^{-1}. If the bullet becomes embedded in the block, determine the initial velocity of the block and the impulse which sets it in motion.

[7 marks]

How much mechanical energy has been lost in the collision? What has happened to it?

[7 marks]

The force acting on the block when it is moving is proportional to the square of its velocity, with the constant of proportionality equal to 4.5×10^{-2} kg m^{-1}. Determine the time after the impact of the bullet at which the velocity of the block will have fallen to one-half of its initial value. What is the acceleration of the block at this time?

[9 marks]

4) Explain the meanings of (a) *damping*, (b) *forced vibration* and (c) *resonance* in an oscillatory system.

[6 marks]

A particle of mass m moving on a smooth horizontal surface is attached to one end of a horizontal massless spring with force constant k; the other end of the spring is attached to a rigid support. The mass is subject to an oscillatory force $F = F_0 \sin wt$ directed along the axis of the spring. By considering the amplitude of the oscillation when the steady state has been reached, show that the resonant frequency of the forced oscillator is equal to the natural frequency $(\sqrt{k/m})$ of the system.

[15 marks]

The surface on which the mass moves is now roughened, so that the mass experiences a frictional damping force proportional to its velocity (proportionality constant = b). The amplitude A of the damped oscillation is now given by $A = \dfrac{F_0}{\sqrt{(k - mw^2)^2 + b^2w^2}}$.

Show that the resonant frequency of the forced damped oscillator is given by $w^2 = \dfrac{2km + b^2}{2m^2}$.

Sketch the dependence of the amplitude A on the driving frequency w for light, intermediate and heavy damping.

[9 marks]

5) What basic postulate underlies the Special Theory of Relativity? What particular consequence does this principle have for the velocity of light?

[7 marks]

Explain how the theory leads to *time dilation* and obtain the relationship $t = t'\sqrt{1-\frac{v^2}{c^2}}$, which describes this phenomenon, taking care to specify clearly the meaning of each symbol in the equation.

[15 marks]

Determine the minimum velocity of muons created at an altitude of 12 km if they are to reach the Earth's surface within their half life. The half-life of muons measured in their own frame of reference is 2.2 μs.

[8 marks]