King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/1400 Classical Mechanics and Special Relativity

Summer 2001

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

Separate answer books must be used for each Section of the paper.

**You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.**

TURN OVER WHEN INSTRUCTED

Acceleration due to gravity	g	$= 9.80 \text{ m s}^{-2}$
Speed of light in free space	c	$= 3.00 \times 10^{8} \text{ m s}^{-1}$
Gravitational constant	G	$= 6.67 \times 10^{-11} \text{ N m}^{2} \text{ kg}^{-2}$
Mass of Earth		$= 6.0 \times 10^{24}$ kg
Diameter of Earth		$= 1.28 \times 10^{4}$ km

SECTION A - Answer **SIX** parts of this section

1.1) Define the term *impulse* as used in mechanics, and state how it is related to momentum. During a match, a tennis player strikes a ball, which has a mass of 28 g, so that its velocity immediately after leaving the racket is $12\,\hat{\mathbf{i}}$ m s^{-1}, where $\hat{\mathbf{i}}$ is a unit vector. The velocity of the ball immediately before it was hit was $-16\,\hat{\mathbf{i}}$ m s^{-1}. If the racket was in contact with the ball for 5 ms, what was the average force exerted by the racket on the ball?

[7 marks]

1.2) A simple pendulum, consisting of a weight attached to a light thread of length l suspended from a rigid support, is set swinging in a vertical plane. Show that, under certain conditions (which should be stated), the pendulum executes simple harmonic motion with a period of $2\pi\sqrt{\dfrac{l}{g}}$.

[7 marks]

1.3) Assuming the Earth to be a uniform sphere, explain why the value of the acceleration of free fall at the north pole of the Earth is different from that at the equator. Determine the difference between the two values.

[7 marks]

1.4) State Kepler's third law of planetary motion. The planets Venus and Neptune in the Solar System have nearly circular orbits with respective radii 1.08×10^{8} km and 4.50×10^{9} km. The orbital period of Venus is 0.62 years. Determine the orbital period of Neptune.

[7 marks]

1.5) Define the term *centre of mass* for a system of particles. Calculate the distance from the centre of the Earth to the centre of mass of the Earth-Moon system.
(Mass of Moon = 7.4×10^{22} kg; radius of Moon's orbit about Earth = 3.8×10^{5} km.)

[7 marks]

1.6) Write down an expression for the energy of a body moving in the Earth's gravitational field. Hence show that the escape velocity of an object at the Earth's surface is 11.2 km s^{-1}.

[7 marks]

1.7) State the physical principle on which the Special Theory of Relativity is based. What particular consequence does this principle have for the speed of light? Two inertial reference frames are moving relative to each other in a direction parallel to their respective x-axes. An observer in one reference frame sees a light signal travelling perpendicular to the x-axis. What is the appearance of the same light signal to an observer in the other reference frame?

[7 marks]

1.8) Distinguish between *fundamental units* and *derived units* in the SI, giving examples of units commonly used in mechanics to illustrate your answer. Modern atomic clocks can measure time to an accuracy of 1 part in 10^{15}. If it were possible to measure length with the same accuracy, express the uncertainty in the diameter of the Earth in terms of a number of carbon atoms, each of which has a diameter of 0.15 nm.

[7 marks]

SECTION B - Answer **TWO** questions

2) Define the terms *moment of inertia* and *angular momentum*.

[6 marks]

A uniform disc has mass M and radius R. Show, from first principles, that the moment of inertia of the disc about an axis through its centre and perpendicular to its face is $\frac{1}{2}MR^2$.

[10 marks]

In the clutch mechanism of a car, the clutch plate is a uniform disc of radius 0.12 m and mass 4.5 kg attached to the transmission shaft. The clutch plate is thrust against the engine flywheel, also a uniform disc, by a force along the common axis of the two discs. The clutch plate is rotating at an angular velocity of 220 rad s^{-1} before it engages with the flywheel. The flywheel has a radius of 0.24 m and a mass of 2.5 kg, and is rotating with an angular velocity of 60 rad s^{-1} before the engagement. If the two discs are rotating freely, and the moments of inertia of their shafts can be neglected, determine the angular velocity of the flywheel and clutch plate after engagement.

[8 marks]

Determine the kinetic energies associated with the clutch mechanism before and after the engagement of the two discs. Is kinetic energy conserved during this process? Give reasons for your answer.

[6 marks]

3) What is meant by (a) a perfectly elastic collision and (b) a totally inelastic collision?

[5 marks]

Consider the following two collision processes:

(i) Two pucks, each having mass m, radius r and speed v, are moving on a collision course along linear, perpendicular trajectories. They are both coated with an impact adhesive, so that at the moment of collision they stick firmly together. At the moment of collision, the line joining the centres of the two pucks makes an angle θ with one of the trajectories. Show that after the collision the two pucks have a velocity of $\frac{v}{\sqrt{2}}$ in a direction that is independent of θ. How much kinetic energy has been lost during the collision? What has happened to it?

[12 marks]

(ii) A smooth puck (A) is moving with a velocity v on an air table, and makes a perfectly elastic collision with a second puck (B), which is stuck firmly to the table. The pucks each have radius r and mass m. The perpendicular distance at the moment of collision between the centre of puck B and the trajectory of puck A through its centre is b (where $b < 2r$). Show that the angular deflection of puck A is $2\sin^{-1}(b/2r)$ and that the change in its momentum is $2mv\sqrt{1-\frac{b^2}{4r^2}}$.

[13 marks]

4) Explain the meanings of (a) *damping*, (b) *forced vibration* and (c) *resonance* in an oscillatory system.

[9 marks]

A particle of mass m moving on a smooth horizontal surface is attached to one end of a horizontal massless spring with force constant k; the other end of the spring is attached to a rigid support. The mass is subject to an oscillatory force $F = F_0 \sin \omega t$ directed along the axis of the spring. By considering the amplitude of the oscillation when the steady state has been reached, show that the resonant frequency of the forced oscillator is equal to the natural frequency $(\sqrt{k/m})$ of the system.

[12 marks]

The surface on which the mass moves is now roughened, so that the mass experiences a frictional damping force proportional to its velocity (proportionality constant = b). The amplitude A of the damped oscillation is now given by $A = \frac{F_0}{\sqrt{(k-m\omega^2)^2 + b^2\omega^2}}$. Show that the resonant frequency of the forced damped oscillator is given by $\omega^2 = \frac{2km + b^2}{2m^2}$.

Sketch the dependence of the amplitude A on the driving frequency ω for light, intermediate and heavy damping.

[9 marks]

SEE NEXT PAGE

5) Define the term *time dilation* in the Special Theory of Relativity. Using a suitable model, derive the relationship $t_{pr} = t_{im}\sqrt{1-\frac{v^2}{c^2}}$, between proper and improper time, taking care to specify clearly the meaning of each symbol in the equation.

[15 marks]

Describe how the Lorentz-Fitzgerald *length contraction* follows from time dilation, and write down the relationship between proper and improper lengths.

[7 marks]

In a test of the Special Theory of Relativity, equipment to count muons arriving at the Earth via the upper atmosphere is set up at two laboratories, one at sea level, the other at a mountain station 2075 m above sea level. In one experiment, muons of velocity 0.994c are counted at the two sites, and it is found that the flux of muons at sea level is reduced compared to that at the mountain laboratory, with the ratio of the two fluxes being 1.3 ± 0.1. Show that this result is consistent with the Special Theory of Relativity. (The half-life of muons in their rest frame is 2.2 μs.)

[8 marks]

FINAL PAGE