King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/1400 Classical Mechanics and Special Relativity

Summer 2000

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A,
and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in s quare brackets.

Separate answer books must be used for each Section of the paper.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Acceleration due to gravity $g = 9.80 \text{ m s}^{-2}$
Gravitational constant $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$

SECTION A - Answer **SIX** parts of this section

1.1) State Kepler's second law of planetary motion. Using a vector representation of the relevant parameters, or otherwise, show that the second law reflects the conservation of angular momentum for an orbiting planet.

[7 marks]

1.2) Draw the world line for a model car that moves in a straight line from rest with a constant acceleration of 1 m s^{-2} for 10 s, maintains the velocity reached for 12 s and is brought abruptly to rest by colliding with a wall.

[7 marks]

1.3) The Mie potential energy for an atom in a diatomic molecule is $U(r) = -\frac{A}{r^m} + \frac{B}{r^n}$, where r is the separation of the atoms and A, B, m and n are positive constants. Show that the equilibrium separation, r_0, of the atoms is given by $r_0^{(m-n)} = \left(\frac{mA}{nB}\right)$.

[7 marks]

1.4) A horizontal spring with one end securely attached to a wall, is stretched by a horizontal force exerted on its free end. The spring constant is 395 N m^{-1} and the spring obeys Hooke's law during the extension. Derive an expression for the work done in stretching the spring and hence calculate the work done in stretching the spring by 55 mm from its natural length.

[7 marks]

1.5) What are the characteristics of a perfectly elastic collision? From the following data relating to a collision process between two spheres, A and B, of the same radius moving on a smooth horizontal surface, determine whether or not the collision is perfectly elastic:

Mass of sphere A, 34 g; mass of sphere B, 68 g. Before the collision, sphere A moves with a velocity of 1.5 m s^{-1} along the line of centres of the spheres, while sphere B is stationary. After the collision, the speeds of spheres A and B are 0.5 m s^{-1} and 1.0 m s^{-1} respectively. (You may assume that no external forces act on the system and that the effects of friction can be neglected.)

[7 marks]

1.6) A mass m is suspended from one end of a vertical spring (spring constant k), the other end of which is fixed to a rigid support. Show that if the mass is displaced vertically from its equilibrium condition it will execute simple harmonic motion of frequency $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

[7 marks]

1.7) What fundamental property of light forms a basis of the special theory of relativity? With the aid of diagrams, but without detailed analysis, show how this property leads to the phenomenon of time dilation.

[7 marks]

1.8) What are the principles by which the fundamental SI units of mass, length and time are established? What is meant by the term *derived unit*? State the names for the derived units of *force*, *energy* and *power*, and express them in terms of the fundamental units.

[7 marks]

SECTION B - Answer **TWO** questions

2) Show, from first principles, that the moment of inertia of a uniform thin rod (mass m, length l) about an axis through its centre and perpendicular to the long axis of the rod is $\frac{1}{12}ml^2$.

[8 marks]

A uniform rod of length 1 m and mass 0.6 kg is freely suspended from one end by a frictionless pivot so that it can rotate in a vertical plane. The rod is held with its axis horizontal and released.

(a) What is the moment of inertia of the rod about the point of suspension?

[5 marks]

(b) Show that the angular acceleration of the rod at the instant immediately after release is $\frac{3g}{2l}$ and determine its value.

[6 marks]

(c) What is the force exerted by the rod on the pivot at this instant?

[6 marks]

(d) Determine the angular velocity of the rod when it reaches its vertical position.

[5 marks]

3) Explain the meaning of the term *centre of mass* when applied to a system of particles.

[4 marks]

Show that if no external forces act on the system, the velocity of the centre of mass is constant.

[6 marks]

If an external force acts on the system, show that the acceleration of the centre of mass is equal to that of a single particle having a mass equal to that of the system and subjected to the same external force.

[5 marks]

Two people, a woman of mass 65 kg and a man of mass 95 kg, separated by 25 m, are standing on ice, so that friction can be neglected, and are holding the ends of a light rope. A puck is placed on the ice midway between them.

(a) Determine the position of the centre of mass of the two people (neglect the mass of the rope).

[3 marks]

(b) The man pulls on the rope so that they approach each other and the puck. Who reaches the puck first? How far is the other person from the puck when the first one reaches it?

[5 marks]

(c) If the man tugs only once on the rope, which they both then release, give a qualitative description of their motion.

[3 marks]

(d) Having released the rope as described in (c), the woman reaches the centre of mass after 4.5 s. What is the man's average speed before he collides with the woman?

[4 marks]

4) A satellite of mass m is in a stable circular orbit of radius r about a planet of mass M, with $M >> m$. Show that the speed of the satellite is $\sqrt{\dfrac{GM}{r}}$, where G is the Gravitational constant. Hence show that the behaviour of the satellite is consistent with Kepler's third law of planetary motion.

[10 marks]

Show that the total energy of the satellite is $-\dfrac{GMm}{2r}$, indicating clearly where the zero for potential energy is taken.

[6 marks]

The mean radius of the Earth's orbit about the sun is 1.5×10^{11} m, while that for Mars is 2.28×10^{11} m. Determine the orbital period for Mars about the sun.

[7 marks]

The length of the Martian day is 1.03 Earth days, while the mass and radius of Mars are 6.42×10^{23} kg and 3.40×10^{6} m respectively. Is it possible to place a satellite in an equatorial orbit to remain above the same point on the Martian surface?

[7 marks]

5) In the special theory of relativity, the Lorentz transformation relates co-ordinates of an event in one reference frame (S) to those of the same event in another reference frame (S') moving along the x-axis with a velocity v relative to the first:

$$x = \gamma(x' + vt'), \quad y = y', \quad z = z', \quad t = \gamma\left(t' + \frac{vx'}{c^2}\right),$$ with $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$, where c is the speed of light.

Show that the velocity transformation equations are:

$$u_x = \frac{u'_x + v}{1 + vu'_x / c^2}, \quad u_y = \frac{u'_y}{\gamma(1 + vu'_x / c^2)}, \quad u_z = \frac{u'_z}{\gamma(1 + vu'_x / c^2)}.$$

[8 marks]

(a) Show that this velocity transformation reduces to the Galilean velocity transformation for $v << c$.

[6 marks]

(b) Show that light rays travelling along (i) the x'-axis and (ii) the y'-axis in the S'-frame have the same speed when measured in the S-frame of reference. In what directions are the light beams travelling in the S-frame?

[8 marks]

A spaceship moves away from you along the x-axis, in the positive x-direction, with a speed of $0.85c$. A second spaceship moves along the x-axis, also in the direction of positive x, away from you and away from the second spaceship at a speed of $0.9c$ relative to the first spaceship. How fast is the second spaceship moving relative to you? What result is expected on the basis of classical mechanics?

[8 marks]

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