SOLAR SYSTEM

EXERCISE SHEET 2

SOLUTIONS

$\boxed{Q2.1}$ (a) Julian date of 12h UTC 16 July 1994:

$$T_1 = 2449550.0$$ (6)

Julian date of 0.6×24 h on 9 February 1986:

$$T_2 = 2446471.1$$ (6)

i.e. days between two dates:

$$\Delta T = 2449550.0 - 2446471.1$$

$$= 3078.9$$ (2)

$a = \dfrac{q}{1-e}$ where q = perihelion distance

i.e. $a = \dfrac{0.5871}{1-0.9673} = 17.9541$ AU (8)

T (orb. period) $= 76$ years (cf. $(17.9541)^{3/2} = 76.076$)

i.e. $n = \dfrac{360}{76 \times 365.25} = 0.0129688$ deg. day^{-1} (8)

i.e. $M = n.\Delta T = 39.9295^\circ = 0.696902$ radians.

(4)

Solving Kepler's equation gives (2)

$$E = 1.66033 \text{ radians} = 95.1298^\circ$$ (16)

$$r = a(1 - e\cos E) = 19.5069 \text{ AU}$$ (4)

(b) orbital energy/unit mass $= \left| -\frac{GM}{2a} \right|$

where G = grav. constant $= 6.672 \times 10^{-11}\ m^3\,kg^{-1}\,s^{-2}$

M = mass of Sun $= 1.98911 \times 10^{30}$ kg

a = semi-major axis of orbits.

i.e. $E_{HALLEY} = 2.47056 \times 10^7\ J\,kg^{-1}$

$E_{SL-9} = 8.52686 \times 10^7\ J\,kg^{-1}$

i.e. $\Delta E = 6.0563 \times 10^7\ J\,kg^{-1}$ (6)

Q2.2 $$C_J = x^2 + y^2 + 2\left(\frac{\mu_1}{r_1} + \frac{\mu_2}{r_2}\right)$$

Rewrite in terms of r_1, r_2 using the fact that

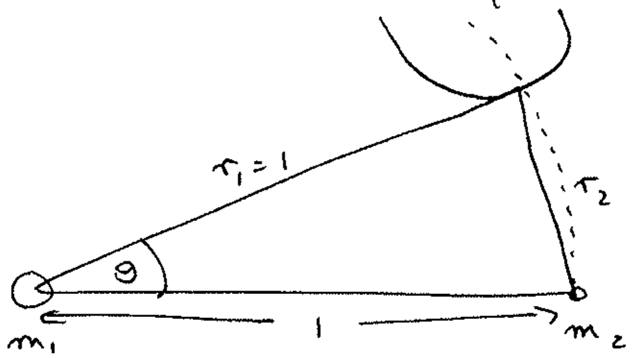
$$\mu_1 r_1^2 + \mu_2 r_2^2 = \mu_1\left[(x+\mu_2)^2 + y^2\right] + \mu_2\left[(x-\mu_1)^2 + y^2\right]$$

$$= \mu_1\left[x^2 + y^2 + 2x\mu_2 + \mu_2^2\right] + \mu_2\left[x^2 + y^2 - 2x\mu_1 + \mu_1^2\right]$$

$$= (x^2+y^2)(\mu_1+\mu_2) + \mu_1\mu_2(\mu_1+\mu_2)$$

$$= x^2 + y^2 + \mu_1\mu_2$$ (6)

i.e. $C_J = \mu_1 r_1^2 + \mu_2 r_2^2 + 2\left(\frac{\mu_1}{r_1} + \frac{\mu_2}{r_2}\right) - \mu_1\mu_2$



(6)

At the critical zero velocity curve going through L_3 we have $C_J = 3 + \mu_2$. Also, $r_1 = 1$ when curve crosses unit circle.

i.e. $C_J = 3 + \mu_2 = \mu_1 + \mu_2 r_2^2 + 2\mu_1 + \frac{2\mu_2}{r_2} - \mu_1\mu_2$

But $\mu_1 = 1 - \mu_2$

i.e. $C_J = 1 - \mu_2 + \mu_2 r_2^2 + 2(1-\mu_2) + \frac{2\mu_2}{r_2} - (1-\mu_2)\mu_2$

$\approx 3 - \mu_2 - 2\mu_2 - \mu_2 + \mu_2 r_2^2 + \frac{2\mu_2}{r_2}$

$= 3 - 4\mu_2 + \frac{2\mu_2}{r_2} + \mu_2 r_2^2$

$= 3 + \mu_2\left(\frac{2}{r_2} + r_2^2 - 4\right)$ (12)

$= 3 + \mu_2$ on the L_3 curve

i.e. $\frac{2}{r_2} + r_2^2 - 4 = 1$ (3)

But, from diagram,

$$r_2 = 2 \sin \frac{\theta}{2} \qquad \text{(isosceles triangle)}$$

i.e. $$\frac{2}{r_2} + r_2^2 - 5 = \left(\sin \frac{\theta}{2}\right)^{-1} + 4 \sin^2 \frac{\theta}{2} - 5 = 0$$

But $$4 \sin^2 \frac{\theta}{2} = 2(1 - \cos\theta)$$

i.e. $$\left(\sin \frac{\theta}{2}\right)^{-1} + 2(1 - \cos\theta) - 5 = 0$$

or $$\left(\sin \frac{\theta}{2}\right)^{-1} - 2\cos\theta - 3 = 0 \qquad (6)$$

This can be ~~reduced~~ solved numerically (Newton-Raphson, etc.) to give $\theta = 23^\circ.9057$ (7)