(d)

If p is the probability of one period ratio lying within $\pm\,\varepsilon_{MAX}$ of a rational, then the probability of N_{OBS} such periods out of N_p periods is

$$P = \binom{N_p}{N_{OBS}} p^{N_{OBS}} q^{N_p - N_{OBS}}$$ (binomial distribution)

$$= \frac{N_p\,!}{(N_p - N_{OBS})!\, N_{OBS}\,!}\, p^{N_{OBS}} (1-p)^{N_p - N_{OBS}}$$ (10)

10 + 10 + 10 + 10 = 40

Q 1.2

$$\frac{1}{2} v^2 - \frac{GM}{r} = C = -\frac{GM}{2a}$$

as $r \to \infty$, $v \to v_\infty$

i.e. at ∞, $C = \frac{1}{2} v_\infty^2 = -\frac{GM}{2a}$

i.e. $\frac{1}{2} v^2 - \frac{GM}{r} = \frac{1}{2} v_\infty^2 = -\frac{GM}{2a}$

i.e. $\underline{a = -\frac{GM}{v_\infty^2}}$ (i.e. $a < 0$) (10)

(2)

at pericentre, $\underline{r_0 = a(1-e)}$

i.e. $e = 1 - \frac{r_0}{a} = 1 + \frac{v_\infty^2 r_0}{GM}$

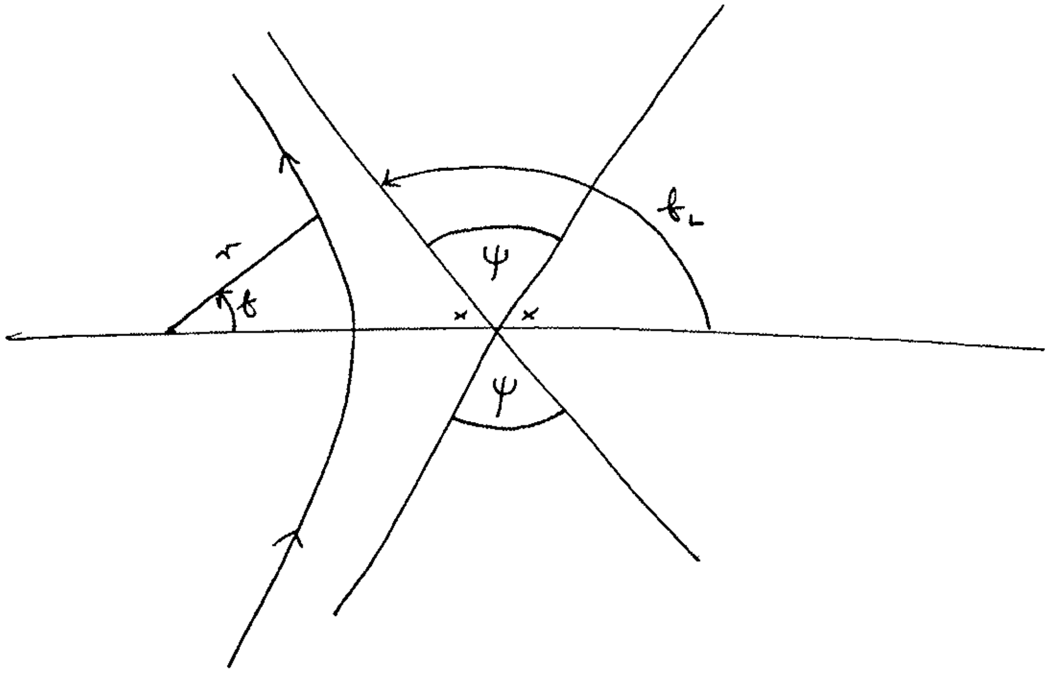
i.e. $\underline{e = 1 + \frac{v_\infty^2 r_0}{GM}}$ (4)

But $v_0^2 = \frac{2GM}{r_0}$ (escape velocity, v_0 at r_0)

i.e. $\dfrac{r_0}{OM} = \dfrac{2}{v_0^2}$

i.e. $e = 1 + \dfrac{2v_\infty^2}{v_0^2}$ (4)

$r = \dfrac{a(1-e^2)}{1+e\cos f}$ where $a<0$, $e>1$



$1 + e\cos f = \dfrac{a(1-e^2)}{r}$ i.e. as $r \to \infty$, $1 + e\cos f \to 0$

i.e. in the limit, $\cos f_L = -\dfrac{1}{e}$

The deflection angle is $\psi = f_L - (\pi - f_L) = 2f_L - \pi$

i.e. $f_L = \dfrac{\psi + \pi}{2}$

i.e. $\cos f_L = \cos\left(\dfrac{\psi}{2} + \dfrac{\pi}{2}\right) = \cos\dfrac{\psi}{2}\cos\dfrac{\psi}{2} - \sin\dfrac{\psi}{2}\sin\dfrac{\pi}{2}$

$= -\sin\dfrac{\psi}{2} = -\dfrac{1}{e}$

i.e. $\sin\dfrac{\psi}{2} = e^{-1}$ (20)

(i) Jupiter $\quad v_0^2 = \dfrac{2GM}{r_0}$

$G = 6.672 \times 10^{-11}\, m^3\, kg^{-1}\, s^{-2} = 6.672 \times 10^{-20}\, km^3\, kg^{-1}\, s^{-2}$

$M = 1898.6 \times 10^{24}\, kg$

$r_0 = 71398\, km$

$$v_0^2 = \frac{2 \times 6.672 \times 10^{-20} \times 1898.6 \times 10^{24}}{71398} = 3548.4073$$

$$\sin\frac{\psi}{2} = e^{-1} = \left(1 + 2\frac{v_\infty^2}{v_0^2}\right)^{-1} = \left(1 + \frac{2 \times 100}{3548}\right)^{-1}$$

$$= 0.9466$$

i.e. $\dfrac{\psi}{2} = 71^\circ.198 \qquad \underline{\psi = 142^\circ.4}$ (10)

(ii) Titan $\quad M = 1345.5 \times 10^{20}\, kg$

$r_0 = 2575\, km$

i.e. $$v_0^2 = \frac{2 \times 6.672 \times 10^{-20} \times 1345.5 \times 10^{20}}{2575} = 6.9726$$

i.e. $$\sin\frac{\psi}{2} = e^{-1} = \left(1 + \frac{2 \times 25}{6.9726}\right)^{-1} = 0.1224$$

i.e. $\dfrac{\psi}{2} = 7.0298\ ; \qquad \underline{\psi = 14^\circ.06}$ (10)