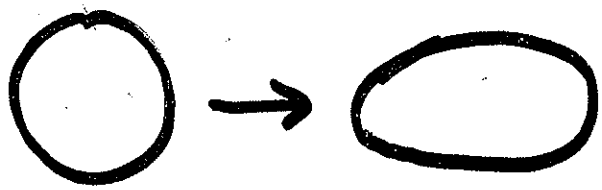
Its angular momentum $L$, is $L \sim MR^2\omega$ and is conserved

$$\Rightarrow \omega \sim \frac{1}{R^2} \Rightarrow \mu_c \simeq \epsilon \sim \frac{1}{R}$$



For the Earth

$$\epsilon = \frac{1}{298.26}$$

# The equation of state

A gravitational bound body is supported by its internal pressure P, which depends on the thermal kinetic energy of its particles.

The equation of a perfect gas is given by

$$P = nkT$$

$n$ is the number density of particles
$k$ is Boltzmann's constant
$T$ is the absolute temperature.
The mass of individual particles is irrelevant here

Since $n = \frac{\rho}{\bar{m}_{mol}}$ ($\bar{m}_{mol}$ mean molecular mass)

$$\bar{m}_{mole} = \frac{\sum_i n_i m_i}{\sum m_i} = A m_p$$ (A mean atomic weight)

Equilibrium condition $\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho(r)$

The state near the centre of a self-gravitating perfect gas can be obtained from this condition.
Pressure gradient can be approximated

$$\frac{dP}{dr} \sim -\frac{P}{R}$$ (P typical pressure)

also $\frac{M(r)}{r^2} \simeq \frac{M}{R^2}$

$\therefore \frac{P}{R} \simeq \frac{GM}{R^2}\rho \simeq \frac{GM}{R^2} A m_p n = \frac{GM}{R^2}\bar{m} n$

compare with the perfect gas law

$\therefore kT \simeq \frac{GM}{R} A m_p$ ( This is a consequence of the Virial theorem

the thermal and the gravitational energies of a particle are of the same order.
This relation holds for the Sun.
The Earth's interior is dominated by inter-molecular forces and is very far from a perfect gas. The gravitational binding energy is shared not only by the three translational degrees of freedom which determine the pressure but also by many other ones.

$$T_\oplus \sim \frac{GM}{kR} A m_p \sim \frac{(\frac{20}{3}\times 10^{-11})(6\times 10^{24})(25)(1.67\times 10^{-27})}{(1.4\times 10^{-23})(6.4\times 10^{6})}$$

$$\sim 1.5\times 10^{5}\ K$$

Actual estimate is $T_\oplus \simeq 4000\ K$.