$$E = \tfrac{1}{2}\dot{\varphi}^2 + 2\omega_0^2 \sin^2 \tfrac{1}{2}\varphi$$

E_{MAX} (energy associated with max. libration) occurs when $\dot{\varphi} = 0$ at $\varphi = \pm\pi$

i.e. $E_{MAX} = 2\omega_0^2 = -6\, j_2^2 C_r\, n\, e^{|j_4|}$

Set E to E_{MAX}:

$$E = \tfrac{1}{2}\dot{\varphi}^2 + 2\omega_0^2 \sin^2 \tfrac{1}{2}\varphi = 2\omega_0^2$$

i.e. $\dot{\varphi}^2 = 4\omega_0^2\left(1 - \sin^2 \tfrac{1}{2}\varphi\right) = 4\omega_0^2 \cos^2 \tfrac{1}{2}\varphi$

i.e. $\dot{\varphi} = \pm 2\omega_0 \cos \tfrac{1}{2}\varphi = \pm j_2 \left(12|C_r|\, n\, e^{|j_4|}\right)^{1/2} \cos \tfrac{1}{2}\varphi$

But $\dfrac{dn}{dt} = 3 j_2 C_r\, n\, e^{|j_4|} \sin\varphi$

i.e. $dn = 3 j_2 C_r\, n\, e^{|j_4|} \sin\varphi \, dt$

$$= 3 j_2 C_r\, n\, e^{|j_4|} \frac{\sin\varphi}{\dot{\varphi}}\, d\varphi$$

$$= \frac{3 j_2 C_r\, n\, e^{|j_4|} \sin\varphi}{\pm j_2 \left(12|C_r|\, n\, e^{|j_4|}\right)^{1/2} \cos\tfrac{1}{2}\varphi}\, d\varphi$$

$$= \frac{6 j_2 C_r\, n\, e^{|j_4|} \sin\tfrac{1}{2} \cos\tfrac{1}{2}\varphi}{\pm j_2 \left(12|C_r|\, n e^{|j_4|}\right)^{1/2} \cos\tfrac{1}{2}\varphi}\, d\varphi$$

$$= \pm\left(3|C_r|\, n\, e^{|j_4|}\right)^{1/2} \sin\tfrac{1}{2}\varphi\, d\varphi$$

i.e. $n = n_0 \pm \left(12|C_r|\, n\, e^{|j_4|}\right)^{1/2} \cos\tfrac{1}{2}\varphi$; $\delta n_{MAX} = \pm\left(12|C_r| n e^{|j_4|}\right)^{1/2}$

Now, from Kepler's 3rd Law,

$$\delta n_{MAX} = -\frac{3}{2}\frac{n}{a}\,\delta a_{MAX}$$

i.e. $$\delta a_{MAX} = -\frac{2}{3}\frac{a}{n}\,\delta n_{MAX}$$

$$= \pm\frac{2}{3}\frac{a}{n}\left(12\,|C_r|\,n\,e^{|j_4|}\right)^{1/2}$$

$$= \pm\left(\frac{16}{3}\frac{|C_r|}{n}\,e^{|j_4|}\right)^{1/2} a$$