$$\frac{d}{dt}\left(v^2\right) = \frac{d}{dt}\left(\dot{\mathbf{r}}\cdot\dot{\mathbf{r}}\right) = \dot{\mathbf{r}}\cdot\frac{d}{dt}\left(\dot{\mathbf{r}}\right) + \frac{d}{dt}\left(\dot{\mathbf{r}}\right)\cdot\dot{\mathbf{r}}$$

$$= \dot{\mathbf{r}}\cdot\ddot{\mathbf{r}} + \ddot{\mathbf{r}}\cdot\dot{\mathbf{r}} = 2\,\dot{\mathbf{r}}\cdot\ddot{\mathbf{r}}$$

Also $\frac{d}{dt}\left(-\frac{\mu}{r}\right) = -\mu\frac{d}{dr}\left(\frac{1}{r}\right)\cdot\left(\frac{dr}{dt}\right) = -\mu\left(-\frac{1}{r^2}\right)\dot{r} = +\frac{\mu\dot{r}}{r^2}$

$\ddot{r} - r\dot{\theta}^2 = -\frac{\mu}{r^2}$ with $u = \frac{1}{r}$

But $\ddot{r} = -h^2u^2\frac{d^2u}{d\theta^2}$ and $-\frac{\mu}{r^2} = -\mu u^2$

i.e. $\ddot{r} - r\dot{\theta}^2 = -h^2u^2\frac{d^2u}{d\theta^2} - \frac{1}{u}\left(hu^2\right)^2 = -\mu u^2$

i.e. $\left(\div \text{ by } -h^2u^2\right)$ $\frac{d^2u}{d\theta^2} + u = \frac{\mu}{h^2}$ (a const.)

Solution $u = \frac{\mu}{h^2}\left[1 + e\cos(\theta - \bar{\omega})\right]$

$$\frac{du}{d\theta} = -\frac{\mu e}{h^2}\sin(\theta - \bar{\omega})$$

$$\frac{d^2u}{d\theta^2} = -\frac{\mu}{h^2}e\cos(\theta - \bar{\omega})$$

$$\frac{d^2u}{d\theta^2} + u = -\frac{\mu}{h^2}e\cos(\theta - \bar{\omega}) + \frac{\mu}{h^2} + \frac{\mu}{h^2}e\cos(\theta - \bar{\omega})$$

$$= \frac{\mu}{h^2} \quad ✓$$