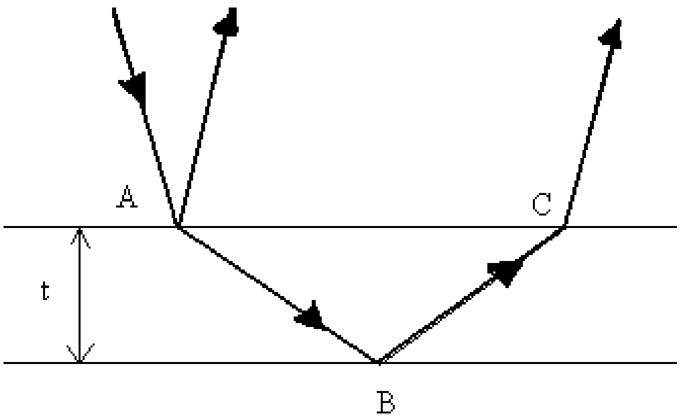
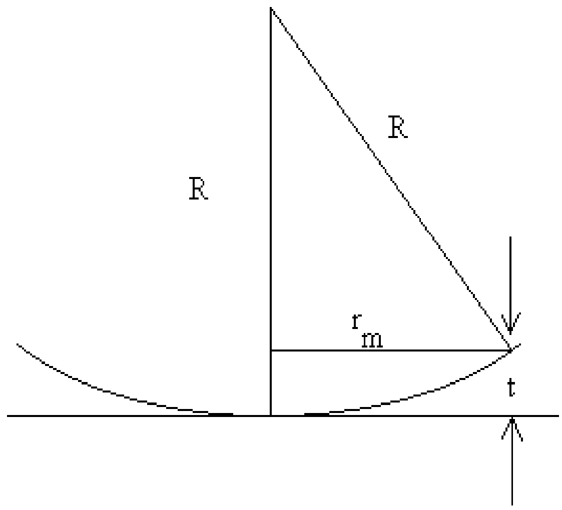
Experiment L18 - Newton's Rings

Introduction:
Newton's rings are interference fringes, which occur when a beam of light is split into two components, which are then recombined after travelling different distances. For example, part of a light beam may be reflected at point A at the top of a transparent medium such as a film, while the remainder passes into the medium. This light is refracted at an angle as it passes in, is reflected off the bottom of the medium at point B and then exits at point C where it is refracted again. The two light waves coming from A and C are now travelling in the same direction, but there is a path difference between them. The two reflected rays will reinforce one another or cancel each other out, depending on the magnitude of their path difference. The amount of refraction undergone by the light will depend on the refractive indices of the film or medium, and its surrounding environment, for example, air.



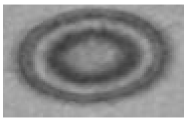
Bright fringes occur when constructive interference occurs and the light waves reinforce each other, i.e. when the path difference = integer × wavelength.
Dark fringes occur when destructive interference occurs and the light waves cancel each other, i.e. when the path difference = (integer + $\frac{1}{2}$) × wavelength.
The path difference depends on the thickness of the film, t, so if the film varies in thickness from place to place a series of fringes of constant thickness will be formed.

Theory:
Newton's rings are formed when light from a source is projected onto a convex lens resting on a uniform glass plate. Interference fringes occur when this light is reflected back from the bottom surface of the lens and the top surface of the glass plate.

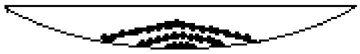


Side view of lens: R is the radius of curvature, r_m is the radius of the *mth* ring.

The interference fringes appear as concentric circles in the bottom of the lens curvature, as the path difference of the reflected light varies due to the lens curvature.

1) 

Above view through a microscope

2) 

Side view (exaggerated drawing) of the lens base

These circular fringes may be used to find the radius of curvature of the lens when light of a known wavelength is beamed down into the lens. In this experiment the diameter of each fringe is measured using a travelling microscope with a Vernier scale.

Objective:
To determine the radius of curvature of a convex lens using a known wavelength of light emitted by a source and the diameters of the circular interference fringes formed when light from a green mercury vapour source is reflected back from the bottom surface up through the lens piece. These circular fringes may be used to determine the constant radius of curvature, because the path difference of the light will vary from place to place as it is reflected back up through the glass lens piece.

Formulae used to calculate the radius of the lens curvature:
The radius of curvature of the lens is related to the radius of the mth ring by the equation:

$$r_m{}^2 = R^2 - (R - t)^2$$

So,

$$r_m{}^2 = t\,(2R - t)$$

But if $t << 2R$ the approximation can be made that:

$$r_m{}^2 = 2\,R\,t$$

For vertical light rays, the path difference is just 2t, so for dark rings:

$$r_m{}^2 = R\,m\,\lambda$$

Since $d^m = 2\,r\,m$, the radius of curvature of the lens, R is related to the diameter of the *mth* ring, d_m by the formula:

$$(d_m)^2 = 4\,R\,m\,\lambda$$ (where m = 1, 2, 3, etc. i.e. the ring number)

R is obtained by measuring the diameters of m consecutive rings formed by the interference fringes and plotting a graph of $(d_m)^2$ against m. R can then be obtained from the gradient of the graph by rearranging the above formula:

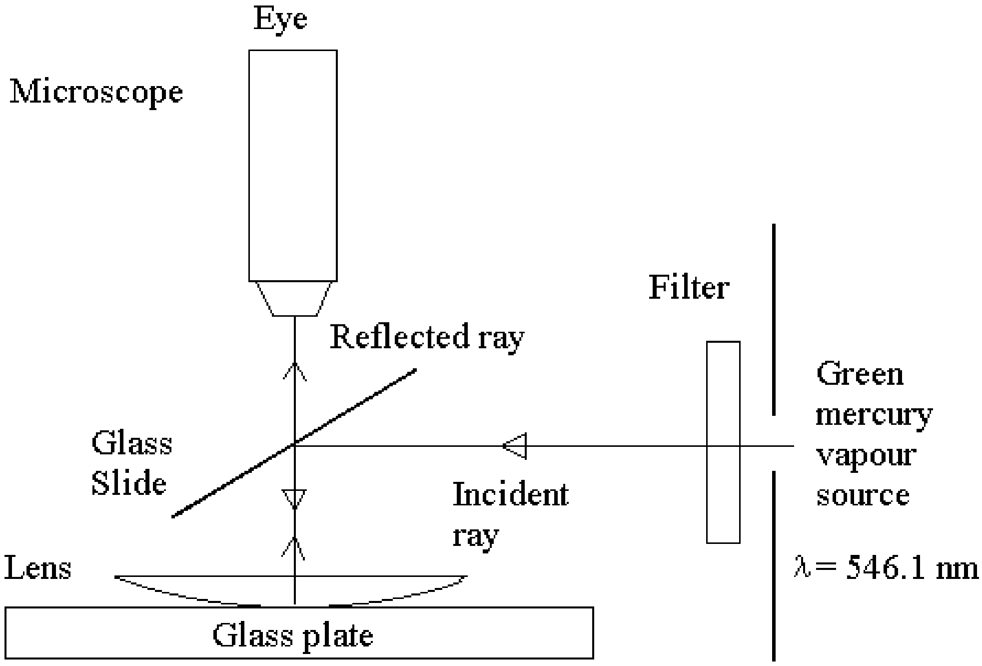
$$\Delta\,(d_m)^2 / \Delta\,m = 4\,R\,\lambda$$

Therefore,
Gradient of graph = $4\,R\,\lambda$

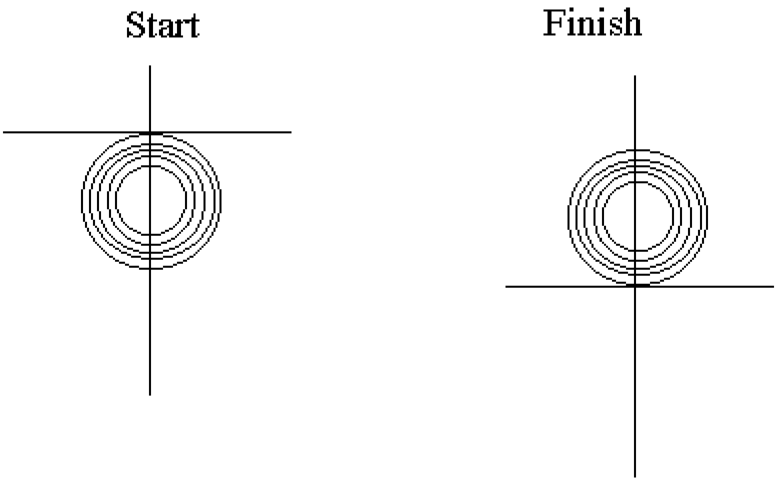
($4\,\lambda$ may be taken as a constant for a known wavelength of light. In this case $4 \times (546.1 \times 10^{-9})$ m = 2.184×10^{-6} m).

Radius of lens curvature, $R = \dfrac{\Delta\,(d_m)^2 / \Delta\,m}{2.184 \times 10^{-6}}$

Method and Experimental set-up:



Green light, of wavelength λ = 546.1 nanometers, is projected from a mercury vapour source down into the lens resting on a glass plate via a glass reflector slide. The interference rings are then viewed through a microscope and the diameters of successive rings measured in such a way as to minimise displacements of the microscope.



The diameters of 18 successive rings were measured by counting outward from the centre of the central 'airy' disk ensuring that the cross hairs of the microscope viewfinder traversed through the centres of the rings. Then scanning inward ring by ring, successive measurements above centre, then below centre were recorded for each of m rings. Measurements were obtained by measuring successive displacements of the microscope against its Vernier scale as it was moved from ring to ring. In this case m = 18 rings. The diameter, d_m is then the modulus of the 'Above centre' measurement minus the corresponding 'Below centre' measurement for each ring.

A graph was then plotted for the (diameter)^2, d_m^2, against ring number, m.

Data and Data Analysis:

The following set of data was obtained for a sample set of Newton's rings viewed and measured under a travelling microscope:

Ring number, m:	**Diameter of ring, d_m (x 10^{-2} m):**	**Diameter ^{2}: $(d_m)^2$ (x 10^{-5} m ^{2}):**
1	0.130	0.1690
2	0.200	0.4000
3	0.240	0.5760
4	0.285	0.8122
5	0.330	1.0890
6	0.350	1.2250
7	0.405	1.6402
8	0.420	1.7640
9	0.440	1.9360
10	0.455	2.0702
11	0.480	2.3040
12	0.510	2.6010
13	0.543	2.9484
14	0.570	3.2490
15	0.590	3.4810
16	0.615	3.7822
17	0.625	3.9062
18	0.650	4.2250

A graph was plotted for this data and a line of 'best fit' was drawn through the data points 'by eye'. The gradient was estimated to be (2.43 ± 0.16) micrometers.

Using the formula:

$$\frac{\Delta(d_m)^2}{\Delta m} = 4R\lambda$$

where λ =546.1 nanometers for the green mercury vapour source

Therefore,

Gradient of the graph = 4 R λ

where $4\lambda = 4 \times (546.1 \times 10^{-9}$ m) = 2.184×10^{-6}, a constant.

From

$$R = \frac{\Delta (d_m)^2 / \Delta m}{2.184 \times 10^{-6}}$$

Substituting in the values obtained for the gradient of the line fitted 'by eye' on the graph,

$$R = \frac{0.243 \times 10^{-5} m^2}{4(546.1 \times 10^{-9} m)} = 1.11243362 \text{ m} \approx 1.112 \text{ m}$$

The uncertainty in the radius of curvature was:

$$\delta R = \frac{0.016 \times 10^{-5} m^2}{4(546.1 \times 10^{-9} m)} = 0.07324666 \text{ m} \approx 0.073 \text{ m}$$

The radius of curvature, R, for the lens piece was then determined to be 1.112 ± 0.073 m based on the gradient of the line of 'best fit'.

A 'least squares' calculation was also carried out for the same data using a computer program ('MSLinefit'). The gradient was calculated to be (2.380 $\pm$ 0.041) micrometers. The radius of curvature was recalculated based on the 'least squares' computer calculation to be (1.090 $\pm$ 0.019) m.

Summary of Calculated Least Squares Data:

I:	**X: (Ring No.)**	**Y: $(d_m)^2$**	**Y CALC:**	**DEVIATION:**
1	1.0000	0.1690	0.0978	-0.0712
2	2.0000	0.4000	0.3358	-0.0642
3	3.0000	0.5760	0.5739	-0.0021
4	4.0000	0.8122	0.8119	-0.0003
5	5.0000	1.0890	1.0499	-0.0391
6	6.0000	1.2250	1.2879	+0.0629
7	7.0000	1.6402	1.5260	-0.1142
8	8.0000	1.7640	1.7640	-0.0000
9	9.0000	1.9360	2.0020	+0.0660
10	10.0000	2.0702	2.2400	+0.1698
11	11.0000	2.3040	2.4781	+0.1741
12	12.0000	2.6010	2.7161	+0.1151
13	13.0000	2.9484	2.9541	+0.0057
14	14.0000	3.2490	3.1921	-0.0569
15	15.0000	3.4810	3.4301	-0.0509
16	16.0000	3.7822	3.6682	-0.1140
17	17.0000	3.9062	3.9062	-0.0000
18	18.0000	4.2250	4.1442	-0.0808

Coordinates of the centre of gravity of the data points:

Mean X = 9.5
Mean Y = 2.121 $\pm$ 0.021

Most Probable Straight Line Through The Data Points:

Gradient m = 0.2380 $\pm$ 0.0041
Y-Intercept c = -0.140 $\pm$ 0.044
X-Intercept b = 0.590 $\pm$ 0.180

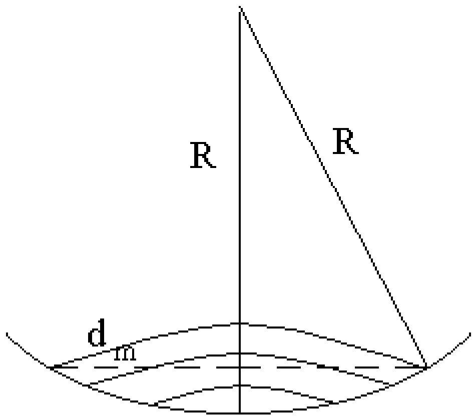
(From computer printout from 'MS Linefit')

The recalculated value of the radius of curvature based on the 'least squares' line of 'best fit' through the data points is:

$$R = \frac{0.2380 \times 10^{-5} m^2}{4(546.1 \times 10^{-9} m)} = 1.089544 \text{ m} \approx 1.090 \text{ m}$$

$$\delta R = \frac{0.0041 \times 10^{-5} m^2}{4(546.1 \times 10^{-9} m)} = 0.018769456 \approx 0.019 \text{ m}$$

Comparison between the two results obtained for the radius of curvature:
The calculated 'least squares' value of 4 R λ has a smaller uncertainty value than the value calculated by fitting a line of best fit 'by eye' whose uncertainty was 4 times as great. The two values themselves were close, however, the value obtained by the 'least squares' method should be closer to the nominal value for the particular lens piece used.



Exaggerated diagrammatic side view of Newton's rings in the bottom of a lens piece. R is the radius of curvature of the lens and d_m is the diameter of the mth ring.

Conclusion:
The two estimates of the radius of curvature of the lens were then compared with the nominal value of 1.0 m:

From the gradient of the line of 'best fit' drawn by eye: R = 1.112 $\pm$ 0.073 m

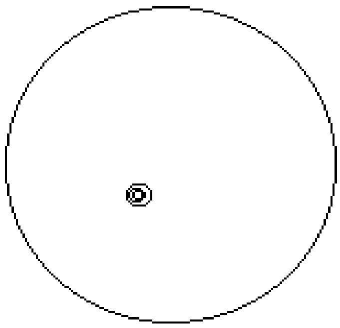
From 'least squares' calculation: R = 1.090 $\pm$ 0.019 m

Nominal value for the radius of curvature for the particular lens used: R = 1.0 m

Therefore the value of the radius of curvature based on the 'least squares' calculation of 4 R λ was closer to the nominal value than the value obtained by fitting a line 'by eye'. Both values were quite close to the nominal value of 1.0 m. The uncertainty in the value of R based on the graph is about four times the uncertainty obtained from the 'least squares' calculation:

0.073 / 0.019 = 3.8421

The least squares calculation of the line of 'best fit' is the more accurate method to use, based on the comparison of the results obtained for radius of curvature and its uncertainty.



Approximation of the relative size of the Newton's rings compared with the overall lens as they appear to the naked eye.