SOLAR SYSTEM

Exercise Sheet 3

Distributed: 25 November, 2010. *Collected*: 9 December, 2010.

3.1 (50%)
The angular momentum per unit mass of an object orbiting the Sun is given by $h = na^2\sqrt{1-e^2}$, where n, a and e are the mean motion, semi-major axis and eccentricity of the object, respectively. Use h to obtain an expression for the angular velocity of the object as a function of the true anomaly, f. An asteroid is in an exact 2:1 resonance with Jupiter. Taking the orbit of Jupiter to be circular, use the result from the first part of this question to show that in a frame rotating with Jupiter's mean motion, the asteroid would be instantaneously stationary at its aphelion provided that its eccentricity e satisfies the equation

$$e^3 + 3e^2 + 7e - 3 = 0\,.$$

Solve this equation to find the critical value of e to four significant figures.

3.2 (50%)
A plot of the distribution of the longitudes of perihelia of the asteroids shows a clustering around the longitude of Jupiter's perihelion. To lowest order in the planar case, the precession rate of an asteroid's longitude of perihelion is given by

$$\dot{\varpi} = \frac{1}{na^2 e}\frac{\partial \mathcal{R}}{\partial e}$$

where n, a and e are the asteroid's mean motion, semi-major axis and eccentricity respectively, and $\mathcal{R}$ is the lowest order secular part of the disturbing function given by

$$\mathcal{R} = \frac{\mathcal{G}m'}{a'}\left\{\frac{A}{8}(e^2 + e'^2) + \frac{B}{4}ee'\cos(\varpi' - \varpi)\right\}$$

where $\mathcal{G}$ is the gravitational constant and m', a', e' and ϖ' are the mass, semi-major axis, eccentricity and longitude of perihelion of Juipter, respectively, and ϖ is the longitude of perihelion of the asteroid; $A = 1.79185$ and $B = -1.18029$ are dimensionless constants. Assuming that all motion is in the plane of Jupiter's orbit (semi-major axis 5.20 AU), derive an expression for $\dot{\varpi}$, the rate of change of the longitude of perihelion of an asteroid orbiting interior to Jupiter. Taking the Jupiter-Sun mass ratio to be 10^{-3}, the eccentricity of Jupiter to be 0.048, and using $a = 2.86$ AU, $e = 0.15$ as typical values for an asteroid, calculate the numerical value of $\dot{\varpi}$ in units of degrees/century in the cases (i) where the perihelia of the asteroid and Jupiter are aligned and (ii) where the perihelia differ by 180°. Use your results to explain the observed distribution of asteroid perihelia.

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