NPA Exam solutions Summer 2008

1) The atomic mass unit, u, is defined such that one atom of carbon-12 has a mass of 12u

2) nuclear radius, r = R.A^(1/3) where R~1.2

3) A=8; Z=5; N=3

4) Mean lifetime is the time taken for the sample to decay to 1/e of its original value and is equal to the inverse of the decay constant \lambda

5) 235U is fissile for all neutron energies, whereas 238U is only fissile for fast neutrons
235U is only ~1% of all naturally occurring U, rest is 238U.
The cross section for 235U(n,f) is 3 orders of mag larger for thermal neutrons, than fast neutrons. Thus reactors require sufficient 235U. In order to sustain the fission reactions enrichment of ~5% is sufficient. This can be achieved by using repeated chains of gas centrifuges to gradually increase the 235U fraction.

lambda_tot = lambda_a + lambda_b

lambda_a = 1/84 = 0.0120
lambda_b = 1/19 = 0.0526
lambda_tot = 0.0120 + 0.0526 = 0.0646 min^-1
mean lifetime, tau = 1/ lambda_tot = 15.5 min

6) Geiger tube: Thin sealed tube contains gas. Inside a single wire anode held at high +ve potential. When ionising radiation enters the tube, gas molecules are ionised. Electrons drift towards the anode and are accelerated. Close to the wire the potential increases as 1/r and an avalanche builds up as primary ionised electrons cause further secondary ionisation in a chain reaction rapidly yielding a charge pulse measured on the wire.

7) Alpha: sharp spikes are observed
Gamma: spikes on a continuum background that falls as E increases
Beta - smooth dist. That falls as E increases, with endpoint at largest E.
It is smooth because energy is shared between beta and unseen neutrino.
Endpoint = Q of reaction, when electron/positron has max energy (neutrino has min energy)

8) false

9) A reaction cross section quantifies the likelihood of a reaction occurring and is measured in terms of an *effective* area. It is not a probability since it is neither dimensionless nor bounded between 0-1

10) X-rays:
high energy photons passed though body. Denser body parts (eg bone) absorb more X-rays than less dense areas (eg muscle). This contrast allows features within the body to be seen
Patient is exposed to damaging ionising radiation

MRI:
Large magnetic field splits two degenerate spin states of hydrogen proportional to B.
Slight overpopulation of lower energy state (1 in 10^9)
RF field excites some protons which decay via emission of photon at this energy
By applying a weak B field in transverse directions with a linear gradient, the splitting becomes spatially dependent.
Measuring the de-excitation photon's energy allows the position to be determined - 3d info obtained.
No known side effects

PET
positron emission tomography
Patient injests a short lifetime beta+ emitter. Emitted positrons annihilate to 2 gammas at 511KeV which are detected
Can map in real time and in 3d
Patient is exposed to damaging ionising radiation

CAT
Pass xrays through body in many directions in turn, and measure intensity loss:
Allows 3d image of body to be made
Patient is exposed to damaging ionising radiation

11) Binding Energy, B(235-U) / 235 = 7.5 MeV:
Total B = 7.5 * 235 = 1762.5 MeV

B(92-Kr) /92 = B(141-Ba) / 141 = 8.5 ;
Total B = 8.5*92 + 8.5*141 = 1980.5 MeV
neglecting the 2 missing neutrons...
total fission energy release = 1980.5 - 1762.5 = 218 MeV

12) Electron capture:
Au(A=195,Z=79) + e → Pt(A=195,Z=78) + neutrino_e

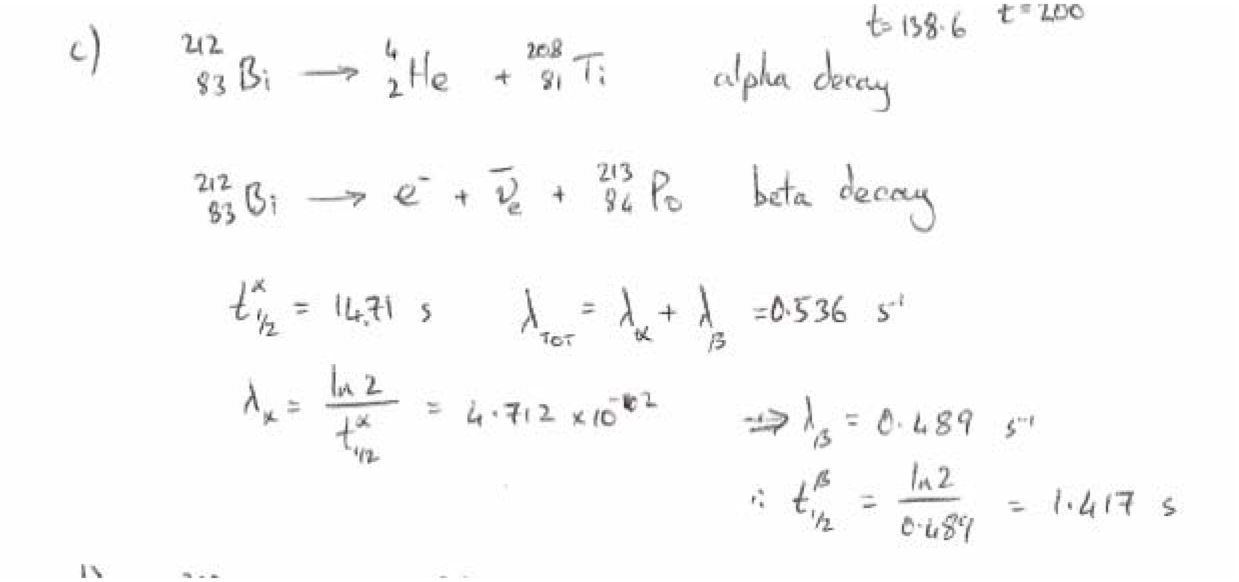
B1)

a) 1 Bq is radioactive material which has on average 1 decay per second

b) lambda_tot = lambda_a + lambda_b
mean lifetime, tau = 1/ lambda_tot
1/tau_tot = 1/lambda_a + 1/lambda_b

or

tau_tot = (tau_a . tau_b) / (tau_a + tau_b)



$t = 138.6$ $t = 200$

c) $^{212}_{83}Bi \longrightarrow {}^{4}_{2}He + {}^{208}_{81}Ti$ alpha decay

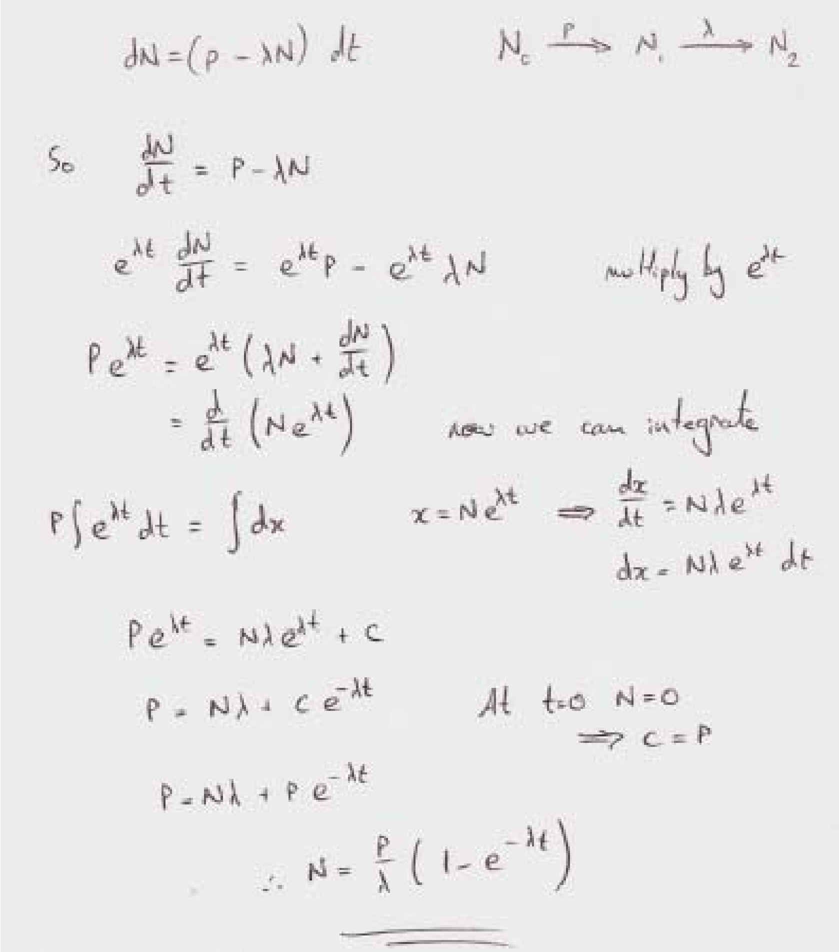
$^{212}_{83}Bi \longrightarrow e^- + \bar{\nu}_e + {}^{213}_{84}Po$ beta decay

$t^{\alpha}_{1/2} = 14.71$ s $\lambda_{TOT} = \lambda_{\alpha} + \lambda_{\beta} = 0.536$ s^{-1}

$\lambda_{\alpha} = \frac{\ln 2}{t^{\alpha}_{1/2}} = 4.712 \times 10^{-2}$

$\Rightarrow \lambda_{\beta} = 0.489$ s^{-1}

$\therefore t^{\beta}_{1/2} = \frac{\ln 2}{0.489} = 1.417$ s d)



$$dN = (P - \lambda N)\, dt \qquad N_0 \xrightarrow{P} N_1 \xrightarrow{\lambda} N_2$$

So $$\frac{dN}{dt} = P - \lambda N$$

$$e^{\lambda t}\frac{dN}{dt} = e^{\lambda t}P - e^{\lambda t}\lambda N$$ multiply by $e^{\lambda t}$

$$Pe^{\lambda t} = e^{\lambda t}\left(\lambda N + \frac{dN}{dt}\right)$$
$$= \frac{d}{dt}\left(Ne^{\lambda t}\right)$$ now we can integrate

$$P\int e^{\lambda t}\, dt = \int dx \qquad x = Ne^{\lambda t} \Rightarrow \frac{dx}{dt} = N\lambda e^{\lambda t}$$
$$dx = N\lambda e^{\lambda t}\, dt$$

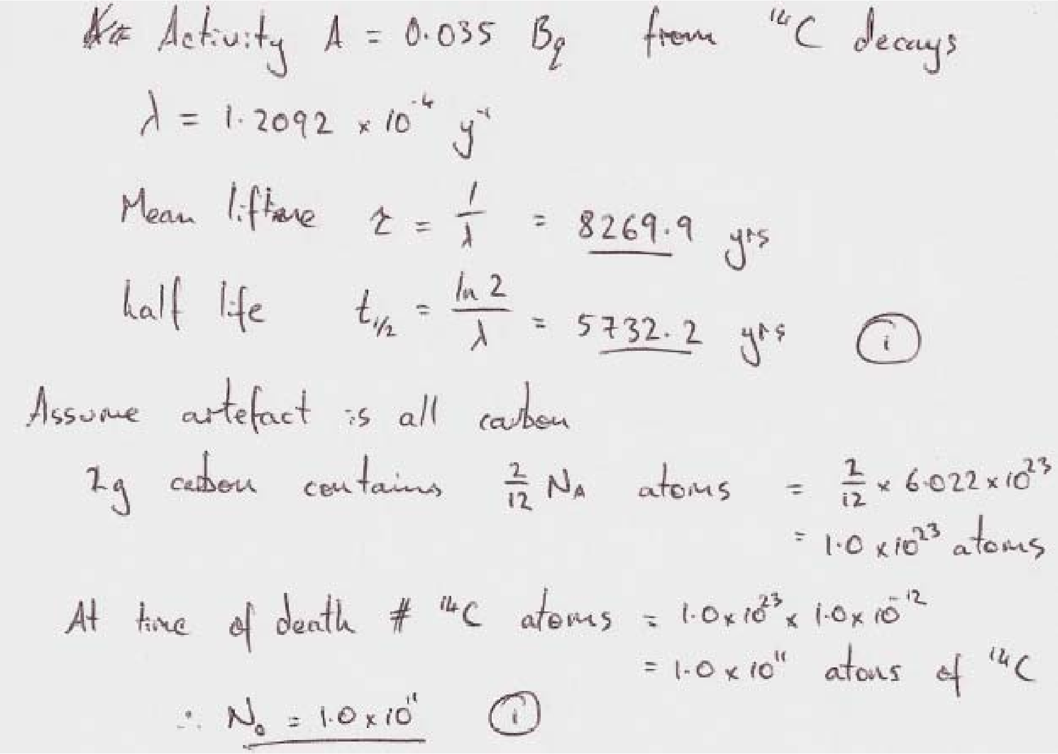
$$Pe^{\lambda t} = N\lambda e^{\lambda t} + c$$

$$P = N\lambda + ce^{-\lambda t}$$ At $t=0$ $N=0$ $\Rightarrow c = P$

$$P = N\lambda + Pe^{-\lambda t}$$

$$\therefore N = \frac{P}{\lambda}\left(1 - e^{-\lambda t}\right)$$

e) carbon dating can be used to estimate the age of organic artifacts. Organic matter absorbs CO2 from atmosphere (or via plant consumptions which in turn absorb CO2 from the atmosphere). Isotope carbon-12 is stable and most abundant isotope (98.89%), remainder is c-13. There is a trace amount of c-14 coming from cosmic ray production in upper atmosphere. Carbon-14 is unstable with half life of ~5000y. During the life of organism c-14 ratio is constant, but when it dies it stops acquiring c-14 which decays slowly. This is used to determine the time when the organism died by measuring the activity.



~~A~~ Activity $A = 0.035\ Bq$ from ^{14}C decays

$\lambda = 1.2092 \times 10^{-4}\ y^{-1}$

Mean lifetime $\tau = \frac{1}{\lambda} = \underline{8269.9}$ yrs

half life $t_{1/2} = \frac{\ln 2}{\lambda} = \underline{5732.2}$ yrs ①

Assume artefact is all carbon

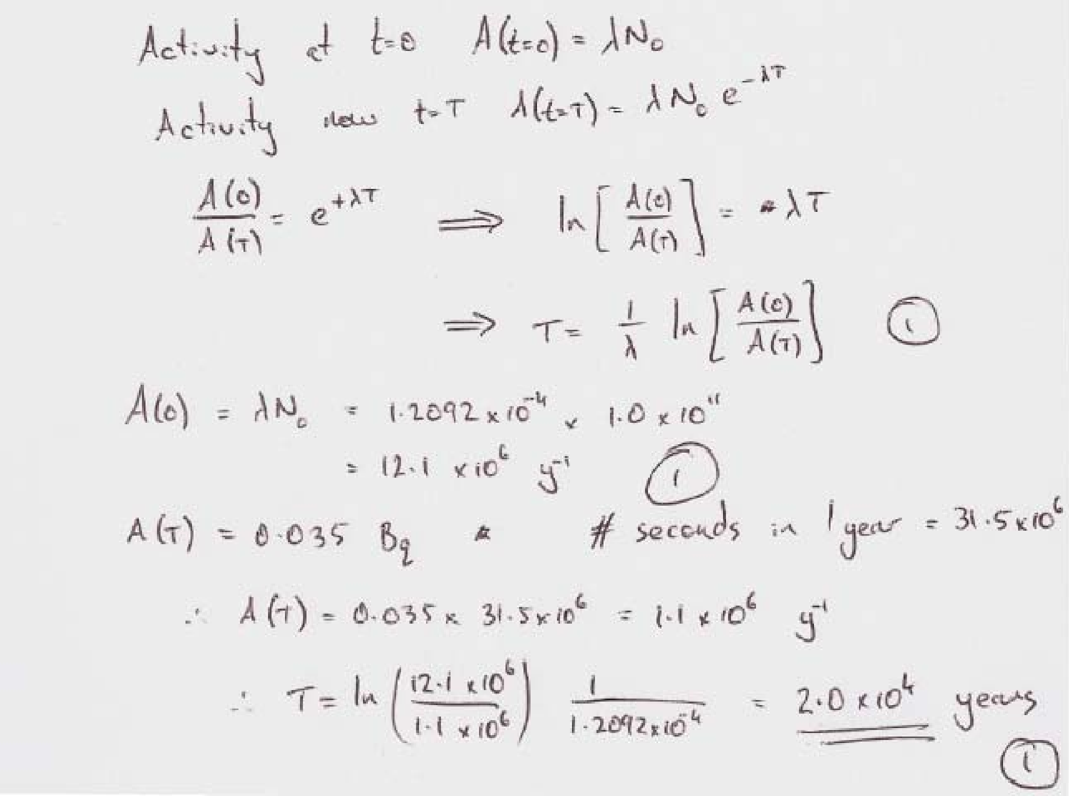
$2g$ carbon contains $\frac{2}{12} N_A$ atoms $= \frac{2}{12} \times 6.022 \times 10^{23}$

$= 1.0 \times 10^{23}$ atoms

At time of death # ^{14}C atoms $= 1.0 \times 10^{23} \times 1.0 \times 10^{-12}$

$= 1.0 \times 10^{11}$ atoms of ^{14}C

$\therefore \underline{N_0 = 1.0 \times 10^{11}}$ ①



Activity at $t=0$ $A(t=0) = \lambda N_0$

Activity now $t=T$ $A(t=T) = \lambda N_0 e^{-\lambda T}$

$$\frac{A(0)}{A(T)} = e^{+\lambda T} \implies \ln\left[\frac{A(0)}{A(T)}\right] = \lambda T$$

$$\implies T = \frac{1}{\lambda} \ln\left[\frac{A(0)}{A(T)}\right]$$ ①

$A(0) = \lambda N_0 = 1.2092 \times 10^{-4} \times 1.0 \times 10^{11}$

$= 12.1 \times 10^{6}\ y^{-1}$ ①

$A(T) = 0.035\ Bq$ # seconds in 1 year $= 31.5 \times 10^{6}$

$\therefore A(T) = 0.035 \times 31.5 \times 10^{6} = 1.1 \times 10^{6}\ y^{-1}$

$\therefore T = \ln\left(\frac{12.1 \times 10^{6}}{1.1 \times 10^{6}}\right) \frac{1}{1.2092 \times 10^{-4}} = \underline{\underline{2.0 \times 10^{4}}}$ years ①

g) The carbon-14 fraction can be affected by the cosmic ray flux which is in turn affected by the sun’s activity. It can also be affected by atomic weapon tests and explosion which may have produced additional carbon-14 since testing began in 1940s.

B2

a) strong; weak; Electromagnetic; gravity
gravity not included in SM as gravity is too feeble compared to other three forces in context of subatomic particles. There is no theory of quantum gravity.

b) 1 2 3 <-----generations
u c t = up; charm; top = +2/3 charge
d s b = down; strange; bottom = -1/3 charge

1	2	3	<---- generations
elec neutrino ;	mu neutrino ;	tau neutrino ;	charge = 0
electron	muon	tau	charge = -1

c)

photon	charge = 0	EM force
W+/-	charge = +/-1	weak force
Z	charge = 0	weak force
gluon	charge = 0	strong force

d) neutron = udd
proton = uud

e)

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 u ----> d + W+
             |
             -------> antilepton + neutrino (eg. positron, mu+, tau+)
d-------> d
u-------> u
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f) Mass M = 80 x 10^3 MeV/c^2
uncertainty princ.: dE . dt ~ hbar / 2
take dE = Mc^2
take dt = R/c   R = range
hbar c = 200 MeV fm
R ~ hbar c / (2 M c^2) ~ 200/(2 . 80 x 10^3) ~ 10^-3 fm
very short range force!

B3)

a) In primordial nucleosynthesis shortly after the Big Bang, (~3 mins) universe is hot enough for deuterium fusion to occur, but cool enough so that photo-dissociation of deuterium does not occur. Fusion to pp or nn states does not occur as these are not bound states. Futher fusion reactions produce 3H, 3He, and 4He and trace amounts of Li, B.
H = 76 %
He = 24%

In stellar burning heavier nuclei form in shells within a star. Heavier elements require hotter temps to overcome the larger Coulomb repulsion and so are found in the cores of stars. Elements upto Fe can be formed in this way after which no further fusion is possible as no more binding energy can be gained.

Heavier elements are produced via chains of beta decays and neutron absorbtion via the r and s processes. The s process (slow) takes place over O(10^4) years as elements absorb neutrons and then undergo beta- decay to form larger Z elements.

The r-process occurs when the neutron flux is very high and happens rapidly when the time time for neutron absorption is very small compared to the beta decay lifetimes. Here the large neutron flux yields long chains of heavier isotopes (eg. Fe56 -> Fe57 -> Fe 58 -> Fe59 etc…) until highly unstable isotopes are formed which then undergo beta- decay increasing Z by 1 and start another long chain of neutron absorption isotopes.

b) Q = sum of initial masses – sum of final masses
   a. d+d -> 3He + n
      Q = 2*m(2H) – m(3He) – m(n)
      = (2 * 2.014102 – 3.016029) * 931.502 – 939.573
      = 3.27 MeV
   b. d+d -> 3H + p
      Q = 2*m(2H) – m(3H) – m(p)
      = (2 * 2.014102 – 3.016049) * 931.502 – 938.280
      = 4.54 MeV

c) The thermal kinetic energy of the deuterons must be at least equal to the electrostatic repulsion of the deuterons if these were ‘spheres’ with surfaces touching.
Deuteron radius r = 1.2*A^(1/3) = 1.5 fm
Coulomb potential = Z1 * Z2 *e^2/(4 \pi \epsilon 2r) Z1 = Z2 = 1
= 1.439976/1.5 = 0.479992 MeV
Thermal kinetic energy = 3/2kT
Thus if each particle has (3/2)kT = (1/2) * 0.479992 MeV
Thus T = (1/3) *0.479992/k
= 1.8 x 10^9 K

d) Q = sum of final kinetic energies – sum of initial kinetic energies
Initial KE (Ti = 0.479992 MeV). Thus Q
Thus Total final KE = Q + 0.479992 MeV
   a.) KE(final) = 3.75 MeV
   b.) KE(final) = 5.02 MeV

e) Ang.Mom quantum is given by the letter:

s	p	d	f	g
0	1	2	3	4

Total spin quantum number given by subscript
Thus 1s(1/2) corresponds to a state with L=0 and j=1/2 and occupancy 2*j+1 = 2
In Independent Particle Model only unpaired nucleon determines spin/parity
Parity P=(-1)^L

a. Lithium-7 has unpaired proton
   This sits in 1p(3/2) shell.
   L = 1
   Thus spin/parity = (3/2) −
b. Carbon-13 has unpaired neutron (N = 13 − 6 = 7)
   This sits in 1p(1/2) shell
   L = 1
   Thus spin/parity = (1/2) −