Measurements And Errors I (8/10/07)

Types Of Error

- Random

Such as fluctuations in the apparatus and surrounding, noise in electrics circuits, inaccuracies in setting up the apparatus and in reading the scales. As these errors are random they average out.

- Systematic

These errors are the same for each reading hence there is no advantage in repetition; examples are zero offsets and calibration errors.

- Mistakes

Quoting A Measurement

- A measurement without an estimated error (or uncertainty) is meaningless. Also numerical values are meaningless without SI units.

- Quote measurements in the form a $\pm$ b, where:
  a is the measurement
  b is the error (given to the same number of d.p.).

  i.e. For the speed of light c = $(2.94 \pm 0.04) \times 10^8$ ms^{-1}

  This means the true value probably lies between 2.90 x 10^8 ms^{-1} and 2.98 x 10^8 ms^{-1}, the probability of this is approximately 68% (i.e. after numerous repeats approximately $\frac{2}{3}$ will lie in the given range.

- Errors by definition are known roughly; quote errors to no more then 2 s.f, although 1 is normally sufficient.

- In an experiment with n independent measurements, x_i are made of the quaintly x, the best estimate of the true value of x is given by the mean:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

  The spread of the measurements is defined by a sample standard deviation, s:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

  N.B (Expression must be square rooted to give the sample standard deviation)

  s is also written as σ_{n-1} and referred to as the standard deviation.

- The best *estimate* if the random error in the mean of n independent measurements, σ_m the Standard Error Of The Mean is given by:

$$\sigma_m = \frac{s}{\sqrt{n}}$$

- If the random error in a particular measurement is the principle source of error, it is usually with repeating this measurement a number of times, to reduce the final error. But systematic errors are not reduced.

- If z = x $\pm$ y
  Then errors add in quadrature $\sigma_z = \sqrt{\sigma_x{}^2 + \sigma_y{}^2}$

- If z= xy, z= x / y
  Then fractional errors add in quadrature $\sigma_z = z\sqrt{(\frac{\sigma_x}{x})^2 + (\frac{\sigma_y}{y})^2}$

- If z = kx
  Then $\sigma_z = nx^{n-1}\sigma_x$

Eliminate systematic errors wherever possible by performing a different experiment that is not subject to this error (in the case of a zero offset error). If this is not possible make a realistic estimate for the error.