Extrasolar Planets and Astrophysical Discs
Problem Set 5: Solutions

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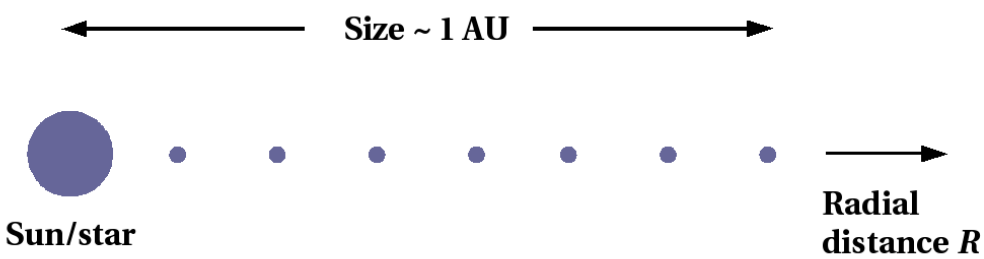
Consider a number of equally spaced protoplanets.



The volume swept out in a time t by a protoplanet moving with a velocity v is $\sigma v t$, where σ is its cross-sectional area.
The collision time is therefore

$$\tau_{coll} \simeq \frac{1}{n\sigma v} ,$$

where v is the velocity dispersion, σ is the cross-section of the individual planets, and n is the number density of the planets.
The velocity dispersion v, the half-thickness H_s of the layer of protoplanets, and their orbital angular velocity Ω are related by

$$v = H_s \Omega .$$

The number density n of protoplanets can be expressed in terms of their number per unit surface area N of the disc by

$$n \simeq \frac{N}{2H_s}$$

time,

$$\tau_{coll} \sim \frac{1}{\left(\frac{N}{2H_s}\right) \sigma \left(H_s \Omega\right)} \sim \frac{2}{N\sigma\Omega}$$

We can express the cross-section as $\sigma = \pi r_p^2$, where r_p is the planet radius.
Since the dispersion velocity must be large for the planet orbits to overlap, the gravitational focussing is not very important and can be ignored here. If the total number of protoplanets with a distance R from the Sun is $n_{planets}$, the average number per unit surface area within R from the Sun is

$$N = \frac{n_{planets}}{\pi R^2}$$

Substituting for N,

$$\tau_{coll} \sim \frac{2}{\left(\frac{n_{planets}}{\pi R^2}\right) \pi r_p^2 \Omega} = \frac{1}{2\pi} \frac{2}{n_{planets}} \frac{R^2}{r_p^2} \frac{2\pi}{\Omega} = \frac{1}{\pi \, n_{planets}} \frac{R^2}{r_p^2} \text{ orbits}$$

because $2\pi/\Omega$ is the orbital period.

To estimate τ_{coll} we shall take $R \simeq 1\text{AU} = 1.5 \times 10^8$ km, $r_p \simeq 0.5 R_{Earth} \simeq 3000$ km and $n_{planets} \simeq$ few. This gives

$$\tau_{coll} \sim \text{few} \times 10^8 \text{ orbits} \sim \text{few} \times 10^8 \text{ years}$$

because the orbital period at 1 AU from the Sun is one year.

Obviously, giant planet cores need to form by a faster mechanism than this so that they can accrete gas from the protostellar disc before it disperses.

Interestingly, detailed computer simulations of the final stages of terrestrial planet formation suggest that it takes 3×10^8 years to form a planetary system that looks similar to the inner Solar System.