Physical Constants

Permittivity of free space	ε_0	=	8.854×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
Speed of light in free space	c	=	2.998×10^{8}	$\mathrm{m\,s^{-1}}$
Gravitational constant	G	=	6.673×10^{-11}	$\mathrm{N\,m^{2}\,kg^{-2}}$
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_e	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_u	=	1.661×10^{-27}	kg
Proton rest mass	m_p	=	1.673×10^{-27}	kg
Neutron rest mass	m_n	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	J s
Boltzmann constant	k_B	=	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ	=	5.670×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
Gas constant	R	=	8.314	$\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_A	=	6.022×10^{23}	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP		=	2.241×10^{-2}	$\mathrm{m^{3}}$
One standard atmosphere	P_0	=	1.013×10^{5}	$\mathrm{N\,m^{-2}}$

Atomic number of carbon	6
Atomic weight of carbon	≈12
Atomic number of oxygen	8

SEE NEXT PAGE

SECTION A – Answer ALL parts of this section

1.1) A plasma consisting of ionised carbon has an electron temperature of 80 eV. Assuming that the electron and ion temperatures are equal, estimate the mean electron and ion speeds.

[7 marks]

1.2) The plasma angular frequency ω_P is given by

$$\omega_\mathrm{P}^2 = \frac{e^2 n_\mathrm{e}}{\varepsilon_0 m_\mathrm{e}}$$

where n_e is the plasma electron density. State the significance of the critical density n_cr and use the equation for ω_P to define it. Calculate a value for n_cr when the plasma is created by a pulsed laser of wavelength 193 nm.

[5 marks]

1.3) State the significance of the *Debye length* λ_D of a plasma, which is given by

$$\lambda_\mathrm{D} = \left(\frac{\varepsilon_0 k_\mathrm{B} T}{e^2 n_\mathrm{e}}\right)^{1/2} .$$

Estimate the Debye length of the Solar core, which has a temperature of $\approx 1.6\times10^7$ K and a density of $\approx 1.6\times10^5\,\mathrm{kg\,m^{-3}}$. Hence estimate the number of particles in the Debye sphere, stating your assumptions. Briefly discuss whether the Solar core should be treated as a collisional or as a collisionless plasma.

[9 marks]

1.4) Under what conditions is *coronal equilibrium* suitable for describing a plasma? What determines the populations of the ionic energy levels in such a plasma? Discuss the transition between coronal equilibrium and local thermodynamic equilibrium.

[6 marks]

1.5) Show that, in the nearest neighbour approximation, the profile of a Stark broadened spectral line with central frequency ν_0 is proportional to $|\nu - \nu_0|^{-5/2}$,. Discuss why, close to ν_0, the profile deviates from this shape.

[7 marks]

1.6) Describe qualitatively, and with the aid of a diagram, the magnetic field structure of a tokamak.

[6 marks]

SECTION B – Answer TWO questions

2) A plasma is formed by focusing a pulsed laser beam onto a Mylar ($C_{10}H_8O_4$) target. The laser beam has a wavelength of 532 nm, a pulse energy of 80 mJ and a pulse length of 800 ps. The diameter of the focal spot is 10 µm.

a) Briefly describe the processes which take place during the formation of the plasma.

[6 marks]

b) Calculate the irradiance in $\mathrm{W\,m^{-2}}$.

[4 marks]

c) Shortly after the start of the laser pulse, the plasma consists predominantly of hydrogen-like ions of carbon (C VI) and oxygen (O VIII). Given that the Lyman α and β spectral emission lines of neutral hydrogen have wavelengths of 121.5670 nm and 102.5722 nm, respectively, determine the wavelengths of the corresponding plasma emission lines.

[4 marks]

d) The C VI Lyman α line is Doppler broadened, with a profile

$$I(\nu) = I(\nu_0)\exp\left[-\frac{(\nu-\nu_0)^2}{2\bar{\beta}_{\|}^2\nu_0^2}\right]$$

where ν_0 is the central line frequency. Define the symbol $\bar{\beta}_{\|}$ and give an expression for it in terms of the ion temperature. Show that the full width at half maximum [FWHM] of the line is given by $\Delta\nu_G = 2\nu_0\bar{\beta}_{\|}(2\ln 2)^{1/2}$. The measured FWHM in terms of wavelength is 0.0004 nm; determine the temperature, in electron volts, of the carbon ions.

[8 marks]

e) Assuming that the oxygen ions and the free electrons have the same temperature as the carbon ions, which will be moving fastest and which slowest? Discuss whether relativistic effects will be important in this plasma.

[4 marks]

f) Describe, qualitatively, how the wavelengths and intensities of the plasma line emission will change after the end of the laser pulse.

[4 marks]

3) a) Describe, with the aid of a suitably labelled energy-level diagram, the processes of *bremsstrahlung* and *recombination radiation*.

[5 marks]

b) Draw a sketch of the classical trajectory of a single electron interacting with a single ion of charge Ze during the bremsstrahlung process. Use the sketch to define the *impact parameter*, b.

[5 marks]

c) The power W radiated at an angular frequency ω from such an encounter is given by

$$\frac{\mathrm{d}W}{\mathrm{d}\omega} \approx \frac{Z^2 e^6}{(4\pi\varepsilon_0)^3} \frac{8}{3\pi m_e^2 c^5} \frac{1}{\beta_e^2 b^2}$$

where β_e is the original electron speed expressed as a fraction of the speed of light. Justify any assumptions made in deriving this equation.

[3 marks]

d) Imagine that a proton makes a similar encounter with the same ion. Write down an expression for the ratio of the power emitted by the proton to that emitted by the electron, and obtain a value for this ratio assuming that the electron and proton are in thermal equilibrium. Hence justify the claim that ion-ion bremsstrahlung may be ignored compared to electron-ion bremsstrahlung.

[8 marks]

e) Show how the equation for $\mathrm{d}W/\mathrm{d}\omega$ may be modified to give the power spectrum $\mathrm{d}P/\mathrm{d}\omega$ for a single electron colliding with a collection of ions of number density n_i,

$$\frac{\mathrm{d}P}{\mathrm{d}\omega} \approx \frac{Z^2 e^6}{(4\pi\varepsilon_0)^3} \frac{16 n_i}{3 m_e^2 c^4 \beta_e} \ln\frac{b_{max}}{b_{min}}$$

where b_{max} and b_{min} are the maximum and minimum allowable impact parameters, respectively. Present brief qualitative arguments why $b_{max} \neq \infty$ and $b_{min} \neq 0$.

[6 marks]

f) The power spectrum may also be written as

$$\frac{\mathrm{d}P}{\mathrm{d}\omega} \approx \frac{Z^2 e^6}{(4\pi\varepsilon_0)^3} \frac{16 n_i}{3 m_e^2 c^4 \beta_e} \frac{\pi}{\sqrt{3}} G .$$

What is a typical order of magnitude for the Gaunt factor, G? State how the Gaunt factor may be used to modify the equation for $\mathrm{d}P/\mathrm{d}\omega$ to take quantum and relativistic effects into account, and for bremsstrahlung from a collection of electrons.

[3 marks]

4) a) Describe, qualitatively, the three components (K, F and E) of the coronal spectrum.

[6 marks]

b) How was the temperature of the corona determined?

[3 marks]

c) The Solar corona has an electron temperature of $\approx 100\,\text{eV}$. Estimate the mean speed of the dominant ions in the corona, stating any assumptions.

[4 marks]

d) Given that the coronal intensity close the Solar surface is about 10^{-5} times that of the Sun itself, that the Solar radius is $\approx 7\times10^{8}\,\text{m}$ and that the (average) Thomson cross section for scattering of light is $\sigma_\text{T} = 6.65\times10^{-29}\,\text{m}^2$, determine the electron density at the base of the corona.

[7 marks]

e) In a one-dimensional plasma the ion number density n_i and speed v_i are related by

$$m_\text{i} n_\text{i}\left(\frac{\partial}{\partial t} + v_\text{i}\frac{\partial}{\partial x}\right)v_\text{i} = -\gamma\left(ZT_\text{e} + T_\text{i}\right)\frac{\partial n_\text{i}}{\partial \text{x}}$$

where $\gamma = 5/3$ is the ratio of specific heats. Which plasma model is used to derive this equation? In which situation is it valid? Use the equation to obtain an expression for the expansion speed of the plasma, stating any assumptions.

[4 marks]

f) Estimate the expansion speed of the solar corona. Discuss whether this is compatible with the value for the mean ion speed obtained in part c) of this question. Explain any discrepancy between the calculated expansion speed and the measured value of $\approx 500\,\text{km}\,\text{s}^{-1}$.

[6 marks]