Speed of light $c = 2.998\times10^{8}\,\mathrm{m\,s^{-1}}$

Boltzmann constant $k = 1.381\times10^{-23}\,\mathrm{J\,K^{-1}}$

Planck constant $h = 6.626\times10^{-34}\,\mathrm{J\,s}$

Proton mass $m_p = 1.673\times10^{-27}\,\mathrm{kg}$

Electron mass $m_e = 9.109\times10^{-31}\,\mathrm{kg}$

Electron charge $e = -1.602\times10^{-19}\,\mathrm{C}$

Electron volt, $1\,\mathrm{eV} = 1.602\times10^{-19}\,\mathrm{J}$

Permittivity of vacuum $e_0 = 8.854\times10^{-12}\,\mathrm{F\,m^{-1}}$

SECTION A – Answer SIX parts of this section

1.1) Give three examples of astrophysical plasmas, two examples of naturally occurring terrestrial plasmas and two examples of man-made plasmas.

[7 marks]

1.2) Determine the electron temperature, in electron volts, of a plasma of temperature 3×10^{6}K. What is the mean free electron speed in this plasma?

[7 marks]

1.3) A charge q in a plasma moves under the influence of a potential $V(r)$ that satisfies the equation

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}V}{\mathrm{d}r}\right)\approx\frac{2e^2n_e}{e_0kT}V(r)$$

where the symbols have their usual meanings. Show, by substitution, that a suitable form for $V(r)$ is $(q/4\pi e_0 r)\exp(-r/l_D)$. Hence find an expression for the *Debye length* l_D.

[7 marks]

1.4) Describe, with the aid of a suitably labelled energy-level diagram, the three main processes by which a plasma emits radiation. Which is likely to be *least* important for a low Z plasma?

[7 marks]

1.5) The shape of a naturally broadened spectral line with a central frequency n_0 is given by the Lorentzian profile

$$I(n)=I(n_0)\frac{1}{1+[2\mathrm{p}(n-n_0)t\,]^2}\,.$$

Derive an expression for the half width at half maximum (HWHM) of this profile. The HWHM of a naturally broadened line is measured to be 0.01 eV. What is the lifetime of the upper ionic level?

[7 marks]

1.6) Under what conditions is *coronal equilibrium* suitable for describing a plasma? What determines the populations of the ionic energy levels in such a plasma?

[7 marks]

1.7) Briefly describe and compare the microscopic, kinetic and fluid model approaches to plasma dynamics.

[7 marks]

1.8) Briefly describe two forms of pre-ignition heating which can be used in a fusion reactor.

[7 marks]

SECTION B – Answer TWO questions

2) a) The rate of change in intensity as radiation passes through a plasma is given by

$$n_{\mathrm{p}}^2 \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{J_w}{n_{\mathrm{p}}^2}\right) = \left(-J_w a_w + j_w\right) \qquad (2.1)$$

Define the terms n_P, J_w, a_w and j_w.

[7 marks]

b) Obtain solutions of equation (2.1) in the limiting cases of (i) no absorption and (ii) no emission (with constant absorption coefficient), and briefly discuss your solutions.

[8 marks]

c) Briefly discuss why the concept of *optical depth* is useful. By defining a suitable source function rewrite equation (2.1) in terms of the optical depth to give the equation of radiative transfer. Determine the mean optical depth of an optically thick plasma.

[9 marks]

d) Assuming that emission can be ignored and that a_w is constant, obtain values for a_w and the mean free path of radiation in an optically thick plasma if the intensity drops by a factor of 5 over a distance of 20 µm. What is the optical depth of a 30 µm thickness of this plasma?

[6 marks]

3) a) Define the term *plasma frequency*, n_P.

[3 marks]

b) Assuming that thermal energy and collisions can be neglected, show that the equation of motion for a displaced electron in a plasma is

$$m_{\mathrm{e}}\ddot{x} + \left(\frac{e^2 n_{\mathrm{e}}}{e_0}\right)x = 0,$$

where the symbols have their usual meanings. Hence derive an expression for n_P and calculate the value of n_P for a plasma with an electron density of $10^{27}\,\mathrm{m}^{-3}$.

[12 marks]

c) By considering the behaviour of the free electrons in a plasma under the influence of an external time-varying electromagnetic field, derive an expression for the plasma refractive index n_P.

[10 marks]

d) Calculate the maximum electron density that can be obtained if a plasma is formed using a pulsed laser with wavelength 532 nm.

[5 marks]

4) a) Describe the processes which take place in the formation of a laser-produced plasma.

[6 marks]

b) Discuss how laser-produced plasmas are used in studies of *inertial confinement fusion*.

[8 marks]

c) A plasma is formed at an irradiance of $2\times10^{18}\,\mathrm{W\,m^{-2}}$ by focusing a laser beam of circular cross section, pulse length 800 ps and energy 80 mJ onto a carbon target with a spherical lens. Estimate the focal spot radius.

[4 marks]

d) The x-ray emission spectrum of this plasma consists mainly of lines from helium-like carbon, plus some from hydrogen-like carbon. Describe, qualitatively, how the spectrum changes if each of the following are gradually reduced while leaving the other parameters constant: (i) the pulse energy, (ii) the pulse length and (iii) the laser wavelength.

[12 marks]

5) a) Describe the origin of the *solar wind* and how it creates a plasma environment close to the Earth.

[10 marks]

b) Draw a diagram of the plasma structure of the Earth's magnetosphere, showing typical electron densities, electron temperatures and magnetic field strengths for each of the magnetosphere regions.

[10 marks]

c) Describe the origin of the *ionosphere* and the *aurora*.

[5 marks]

d) Discuss the effect on the ionosphere and the aurora if the Earth was slightly closer to the Sun

[5 marks]

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