King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP3402 Solid State Physics

January 2006

Time allowed: THREE Hours

Candidates should answer all SIX parts of SECTION A,
and no more than TWO questions from SECTION B.
No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Physical Constants

Permittivity of free space	ε_0	=	8.854×10^{-12}	$\mathrm{F\ m^{-1}}$
Permeability of free space	μ_0	=	$4\,\pi \times 10^{-7}$	$\mathrm{H\ m^{-1}}$
Speed of light in free space	c	=	2.998×10^{8}	$\mathrm{m\ s^{-1}}$
Gravitational constant	G	=	6.673×10^{-11}	$\mathrm{N\ m^{2}\ kg^{-2}}$
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_e	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_u	=	1.661×10^{-27}	kg
Proton rest mass	m_p	=	1.673×10^{-27}	kg
Neutron rest mass	m_n	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	J s
Boltzmann constant	k_B	=	1.381×10^{-23}	$\mathrm{J\ K^{-1}}$
Stefan-Boltzmann constant	σ	=	5.670×10^{-8}	$\mathrm{W\ m^{2}\ K^{-4}}$
Gas constant	R	=	8.314	$\mathrm{J\ mol^{-1}\ K^{-1}}$
Avogadro constant	N_A	=	6.022×10^{23}	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP		=	2.241×10^{-2}	$\mathrm{m^{3}}$
One standard atmosphere	P_0	=	1.013×10^{5}	$\mathrm{N\ m^{-2}}$

SECTION A – Answer all SIX parts of this section

1.1 Describe the *face centred cubic* (FCC) crystal structure.
Assuming the atoms behave like hard spheres in close contact, calculate the fraction of a FCC crystal that is occupied by atoms.

[7 marks]

1.2 Bragg diffraction of X-rays is observed at an angle of 88º from a certain crystal at a temperature of 20 ºC. When the temperature is raised to 100 ºC the angle of diffraction is 86.2º. Calculate the fractional change in lattice constant per ºC.

[6 marks]

1.3 The heat capacity of a crystalline material was measured at 1 K and 4 K, and found to be 1.36 mJ mol^{-1} and 7.23 mJ mol^{-1}, respectively. Determine, giving a brief explanation of the procedure used, whether the material is a metal or a non-metal. Use sketches, where appropriate.

[6 marks]

1.4 The current density for a metal is $\mathbf{j} = \dfrac{ne^2\tau}{m_e}\mathbf{E}$, where n is the electron concentration, τ is the relaxation time and **E** is the electric field.
Calculate τ for copper at room temperature. Copper is monovalent with a resistivity of 1.7×10^{-8} Ωm at room temperature. The atomic mass number of copper is 63.5 and the density is 8920 kg m^{-3}.

[8 marks]

1.5 Describe briefly the valence band to conduction band absorption processes that occur in an indirect gap semiconductor, and discuss how they vary with temperature.

For an indirect gap semiconductor, a simple model shows that the absorption coefficient α is proportional to $(h\nu - h\nu_{min})^2$, where $h\nu$ is the photon energy, and $h\nu_{min}$ is the lowest photon energy at which a given absorption process can occur.

Sketch this behaviour at temperature $T = 0$ and for $T > 0$.

[8 marks]

1.6 At room temperature the electron and hole concentrations in intrinsic silicon are both equal to 6.7×10^{15} m^{-3}.

Calculate the hole concentration at room temperature in n-type silicon doped with 10^{20} m^{-3} donors.

[5 marks]

SECTION B – Answer TWO questions

2 (a) Explain the meanings of the terms in the expression for the structure factor F_{hkl} in relation to the diffraction of X-rays from a crystalline solid:

$$F_{\mathrm{hkl}} = \sum_j f_j \exp\{2\pi \mathrm{i}\,(h\,x_j + k\,y_j + l\,z_j)\}.$$

[3 marks]

For a given angle of diffraction, on what does the parameter f_j principally depend?

[2 marks]

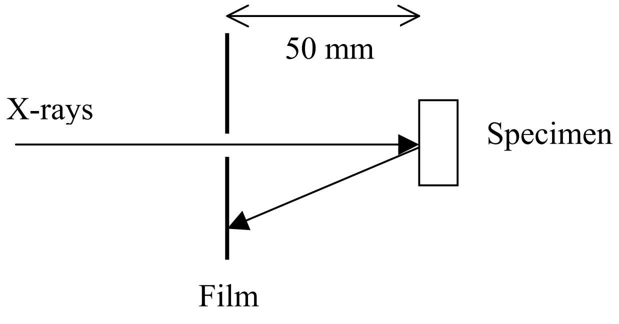
(b) Write down the structure factor for the diamond crystal structure and hence show that space group extinctions occur unless (i) h, k, l are all odd or (ii) h, k, l are all even with $h + k + l = 4m$ where m is an integer. (Hint: be careful to analyse all possibilities when h, k and l are all even.)

[13 marks]

(c) An X-ray examination of a large lump of polycrystalline diamond was carried out using the Laue geometry (see diagram, below). An X-ray beam with wavelength 0.205 nm was directed through the hole in the centre of a sheet of film, and the backward-diffracted X-rays produced a single “ring” on the film. The distance between the film and the specimen was 50 mm.

Calculate the diameter of the ring on the film. (The lattice constant of the cubic unit cell for diamond is a = 0.3567 nm.) Hint: it will be helpful to draw up a table showing all allowed diffractions.

[12 marks]



3. In a one-dimensional chain, atoms of masses M and m alternate, and are coupled together by springs with a force-constant K. Atoms of the same type are separated by a distance a.

The relative phase and amplitude of the vibrations of the atoms are described by a complex number α, and, assuming only nearest-neighbour forces are important, it is straightforward to show that:

$$\alpha = \frac{2K\cos(ka/2)}{2K - \omega^2 m} = \frac{2K - \omega^2 M}{2K\cos(ka/2)}$$

where ω is the angular frequency and k is the wavenumber.

(a) Show that

$$\omega^2 = \frac{K(M+m)}{Mm}\left\{1 \pm \left[1 - \frac{4Mm}{(M+m)^2}\sin^2(ka/2)\right]^{1/2}\right\}.$$

[8 marks]

(b) Show that, as $k \to 0$, there is

(i) a high-frequency mode with $\omega^2 \approx \dfrac{2K(M+m)}{Mm}$ where the atoms with different masses oscillate in antiphase, and

(ii) a low-frequency mode with $\omega^2 \approx \dfrac{Kk^2a^2}{2(M+m)}$ where the atoms oscillate with the same amplitude and phase.

[8 marks]

(c) Sketch the relationships between ω and k for $-2\pi/a < k < 2\pi/a$, and explain why the upper branch is called the *optical branch* and the lower branch is called the *acoustic branch*.

[7 marks]

(d) Aluminium nitride exhibits intense absorption at a wavelength of 15 μm in the infrared spectral region. Assuming that the formulae derived for a linear chain apply approximately to a three-dimensional crystal, estimate the force constant for the ionic bond. The atomic mass numbers of Al and N are 26.98 and 14.01, respectively.

[7 marks]

4. (a) Explain what is meant by an *abrupt p-n junction*. Explain why there is a potential difference built up between the p and n regions.

[5 marks]

(b) Show that the current I flowing through a p-n junction at a bias voltage V is $I = I_0\,[\exp(eV/k_BT) - 1]$, where I_0 is the reverse saturation current.

[10 marks]

(c) Describe briefly the operation of a light-emitting diode, and explain why direct gap semiconductors are required for the production of laser diodes. Give one example of a material suitable for laser emission in the red spectral region, and one suitable for laser emission in the blue.

[6 marks]

(d) At 293 K a certain silicon p-n junction rectifier has a reverse saturation current of 10^{-10} A, and the combined resistance of the material on either side of the junction is 0.05 Ω. Calculate, for forward bias, the current flowing when there is a voltage of 1.5 V dropped across the whole device. (At 293 K, $e/k_BT = 40$.)

[9 marks]