King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CP/3270 Chaos in Physical Systems

Summer 2002

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper. Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

SECTION A – Answer SIX parts of this section

1.1) Determine the fixed point for the system

$$\dot{x} = rx + 4x^3$$

for $r > 0$.

Show that it is unstable.

[7 marks]

1.2) Use dimensionless variables to derive conditions under which it is valid to approximate the equation

$$mL^2\ddot{\theta} + b\dot{\theta} + mgL\sin\theta = \Gamma$$

by

$$b\dot{\theta} + mgL\sin\theta = \Gamma.$$

[7 marks]

1.3) Determine the fixed points of the system

$$\dot{x} = -x + x^3,$$
$$\dot{y} = -2y$$

and use linearization to classify them.

[7 marks]

1.4) Define the term 'gradient system'.

Prove that closed orbits are impossible in gradient systems.

[7 marks]

1.5) Given the Lorenz equations

$$\dot{x} = \sigma(y - x),$$
$$\dot{y} = rx - y - xz,$$
$$\dot{z} = xy - bz$$

show that the z-axis is an invariant line (i.e. the motion confined to the line $x = y = 0$ is permitted).

[7 marks]

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