King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/2481 Electromagnetism and Optics 1

Summer 1998

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Permittivity of free space	ϵ_0	$= 8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	$= 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Speed of light in free space	c	$= 2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$

SECTION A – Answer SIX parts of this section

1.1) Define the *polarisation* $\mathbf{P}$ in the context of the electrostatics of material media. State the equation which relates the electric displacement $\mathbf{D}$, the field strength $\mathbf{E}$ and the polarisation $\mathbf{P}$, and give the SI units of $\mathbf{D}$, $\mathbf{E}$ and $\mathbf{P}$.

[7 marks]

1.2) Derive the *boundary conditions* which apply to the normal and tangential components of the electric field strength $\mathbf{E}$ at the interface between two media with permittivities ε_1 and ε_2.

[7 marks]

1.3) State the integral form of the Ampère law in magnetostatics, illustrating your answer by an appropriate diagram.
Starting with this integral equation, show that the curl of the magnetic field strength $\mathbf{H}$ is equal to the current density $\mathbf{j}$.

[7 marks]

1.4) Magnetic domains and the phenomenon known as hysteresis occur in ferromagnetic materials. Describe what is meant by the terms *magnetic domain* and *hysteresis.*

[7 marks]

1.5) State Maxwell's electromagnetic equations, defining all of the symbols involved.

[7 marks]

1.6) State briefly the *Huygens-Fresnel principle.* Show how it can be used to understand the diffraction of light into the geometric shadow regions beyond an aperture or obstacle.
Identify one difficulty with the Huygens-Fresnel principle.

[7 marks]

1.7) The Fraunhofer diffraction pattern from a regular array of identical apertures can be written as the product of three terms. Describe the physical significance of each term.
Which of these terms is affected when the apertures are located randomly? Describe how the overall appearance of the diffraction pattern changes as a result.

[7 marks]

1.8) Explain briefly whether the following statements are true or false.

a) The diffracted intensity in the Fraunhofer diffraction pattern of a transparent aperture is directly proportional to the area of the aperture.

b) The Fresnel diffraction pattern from a circular aperture will begin to resemble the Fraunhofer diffraction pattern from a circular aperture as the size of the aperture is reduced.

c) The Fraunhofer diffraction pattern from a transparent circular aperture is identical in all respects to the Fraunhofer pattern produced by an opaque circular disc of the same radius.

[7 marks]

SECTION B – Answer TWO questions

2) Define the *electric susceptibility* χ of a dielectric material and state the relation between this quantity and the *relative permittivity* of the material.

[6 marks]

A capacitor is formed of two coaxial metal cylinders each of length l. The inner cylinder has radius a and the outer cylinder has radius b. A charge Q is placed on the inner cylinder.
The space between the cylinders is filled with an inhomogeneous dielectric whose relative permittivity varies with distance R (measured from the axis of symmetry).
Use the Gauss law to derive expressions for the electric displacement $\mathbf{D}$, the electric field strength $\mathbf{E}$ and the polarisation $\mathbf{P}$ of the dielectric as functions of R, neglecting end effects.

[6 marks]

Show that the capacitance of this capacitor is given by

$$C = \frac{2\pi\varepsilon_0 l}{\int_a^b \frac{dR}{R\varepsilon_r(R)}}$$

[8 marks]

If $l = 100\,\text{mm}$, $a = 5\,\text{mm}$, $b = 10\,\text{mm}$ and it is given that

$$\varepsilon_r(R) = b^2/R^2,$$

show that the capacitance is 14.8 pF.

[6 marks]

If $Q = 1\,\text{nC}$, calculate the surface density of polarisation charge on the inner and outer surfaces of the dielectric.

[4 marks]

3) Give a brief account of the phenomenon of *diamagnetism*.

[5 marks]

State what is meant by the term *Amperian current* in relation to the magnetic properties of material media. Derive the equations $\mathbf{J}_A = \mathbf{M}\times\mathbf{n}$ and $\mathbf{j}_A = \nabla\times\mathbf{M}$ for the surface and volume Amperian current densities of a body with magnetisation $\mathbf{M}$. ($\mathbf{n}$ is a unit outward normal to the surface).

[9 marks]

Draw a sketch illustrating the distribution of Amperian currents for a rod of material which is *uniformly* magnetised along its length.

[2 marks]

A wire carrying a current I is centrally located within a cylindrical iron conduit of relative permeability μ_r that has inner radius a and outer radius b. If the radius of the wire is c, use Ampère's circuital law to obtain expressions for the magnetic field strength $\mathbf{H}$ for $c < r < \infty$, where r is the distance from the wire axis.

[4 marks]

Hence write down expressions for the magnetic flux density $\mathbf{B}$ for $c < r < \infty$ and show that the magnetisation $\mathbf{M}$ of the iron is given by

$$\mathbf{M} = \hat{\phi}\frac{(\mu_r - 1)I}{2\pi r}$$

where $\hat{\phi}$ is a unit vector in the ϕ direction.

[4 marks]

Derive formulas for the Amperian current densities $\mathbf{J}_A$ and $\mathbf{j}_A$, given that, for this problem,

$$\nabla \times \mathbf{M} = \hat{z}\frac{1}{r}\frac{\partial}{\partial r}\left(rM_\phi\right).$$

where $\hat{z}$ is a unit vector in the z direction.

[6 marks]

4 (a) Consider an electromagnetic wave field *in free space.* Starting with Maxwell's equations for curl **E** and curl **H**, show that **E** and **H** are solutions of the wave equation

$$\nabla^2 \mathbf{F} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{F}}{\partial t^2}$$

where **F** represents **E** or **H**.
You may assume the formula $\nabla \times \nabla \times \mathbf{F} = \nabla\nabla \cdot \mathbf{F} - \nabla^2 \mathbf{F}$

[7 marks]

If **F** is independent of the Cartesian coordinates y and z, show that the solution of this equation is

$$\mathbf{F} = \mathbf{F}_1(x - vt) + \mathbf{F}_2(x + vt)$$

where $\mathbf{F}_1$ and $\mathbf{F}_2$ are any functions of their respective arguments and v is a constant.
Discuss the physical significance of the parameter v and the two functions $\mathbf{F}_1$ and $\mathbf{F}_2$.

[8 marks]

(b) Write down an expression for the chromatic resolving power of a regular linear diffraction grating with N slits, defining all the symbols used.

[3 marks]

Figure 4.i shows a plot of the diffracted intensity from such a grating, which has N identical slits of width a that have a centre-to-centre separation of $d = 350\,\mu$m. Using the information on the figure, determine:

i) the number N of apertures in the array,

ii) the width a of each of the apertures in the array,

iii) whether the 5th-order principal maximum could be used to resolve two sharp lines with wavelengths of 520 nm and 550 nm.

[12 marks]

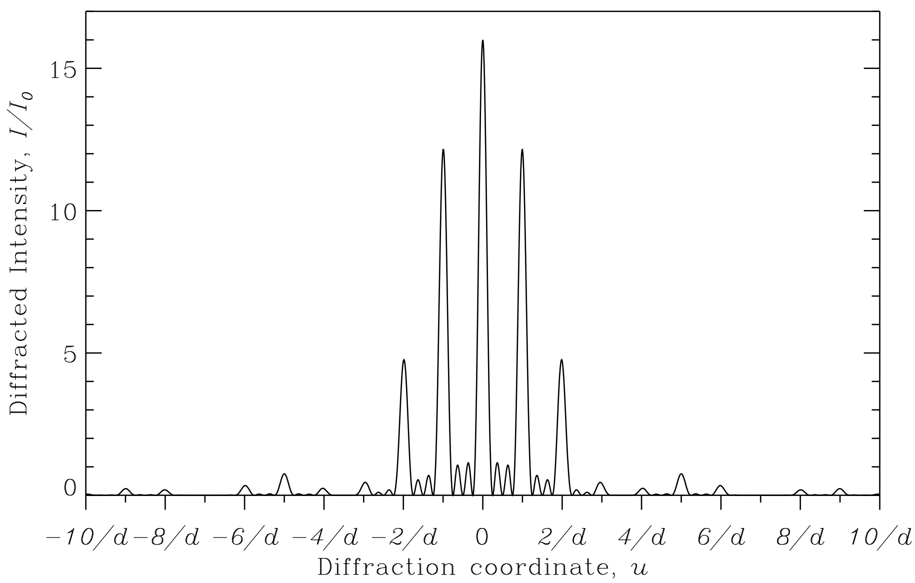

Figure 4.i

5 (a) Briefly distinguish between the conditions needed to observe Fraunhofer and Fresnel diffraction.

[4 marks]

Write down an expression for the far-field diffracted intensity from an object with transmission function $f(x, y)$ when it is illuminated by a monochromatic plane wave propagating along a normal to the object plane. By considering a second object with transmission function $1 - f(x, y)$, deduce Babinet's principle, as it applies to the far-field diffraction patterns from complementary apertures.

[8 marks]

(b) An object is illuminated with a monochromatic plane wave of wavelength $\lambda = 550\,\mathrm{nm}$, and the diffracted intensity in the far field is found to have the form

$$I(u, v) = I_0 \left(\frac{\sin(\pi a u)}{\pi a u} \right)^2 \left(\frac{\sin(4\pi a v)}{4\pi a v} \right)^2 \left(\frac{\sin(20\pi a u)}{\sin(2\pi a u)} \right)^2$$

where u and v are mutually perpendicular axes. Without carrying out a detailed calculation, explain clearly the deductions you can make about the general form of the diffracting object.

[10 marks]

The pattern is observed on a screen 5 m from the object. Calculate the distance between consecutive principal maxima when the object is illuminated with a monochromatic plane wave of wavelength 550 nm, and a is 0.5mm.

[8 marks]