# King's College London

UNIVERSITY OF LONDON



B.Sc. EXAMINATION

CP/2481 Electromagnetism and Optics 1

Summer 1997

Time allowed: THREE Hours

**Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.**

**Separate answer books must be used for each Section of the paper.**

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

**TURN OVER WHEN INSTRUCTED**

| | | |
|---|---|---|
| Permittivity of free space | $\epsilon_0$ | $= 8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$ |
| Permeability of free space | $\mu_0$ | $= 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$ |
| Speed of light in free space | $c$ | $= 2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$ |

## SECTION A – Answer SIX parts of this section

1.1) What is the relation between the polarisation $\mathbf{P}$, the electrical susceptibility $\chi$, and the field strength $\mathbf{E}$ (within the dielectric), for a linear isotropic medium? Explain why the value of $\chi$ is generally larger for polar dielectrics than it is for non-polar dielectrics.

[7 marks]

1.2) State the integral form of the Gauss law as it applies in electrostatics.
A metal sphere of radius $a$ is given an electrical charge $Q$. If the sphere is located in an infinite dielectric medium of relative permittivity $\varepsilon_r$, derive an expression for the electric field strength $\mathbf{E}$ at a distance $r$ from the centre of the sphere.

[7 marks]

1.3) In magnetostatics it is often possible to define a *scalar* magnetic potential $V_m$ so that the magnetic field strength $\mathbf{H}$ is given by the formula $\mathbf{H} = -\nabla V_m$. What essential condition must be satisfied in order for this to apply?
Also explain why this formula cannot be applied if $\mathbf{H}$ varies with time.

[7 marks]

1.4) Derive the *boundary conditions* which must be satisfied by a magnetostatic field $\mathbf{H}$,$\mathbf{B}$ on the interface between two media.

[7 marks]

1.5) The electric field $\mathbf{E}$ and the magnetic field $\mathbf{H}$ of an electromagnetic wave in free space are solutions of the same second-order differential equation. State this equation and its general solution and discuss briefly the physical significance of the general solution.

[7 marks]

1.6) A regular linear array of identical apertures is illuminated by a plane wave of monochromatic light. The diffracted intensity in the far-field can be written as the product of a constant, and two other factors, commonly referred to as the 'diffraction envelope' and the 'interference term'. Explain briefly the physical significance of these two factors.

[7 marks]

1.7) A small iris diaphragm is illuminated by monochromatic light and a diffraction pattern is observed some distance from the aperture. Suggest two ways in which it would be possible to determine whether the observed diffraction pattern is more likely to be Fresnel-like or Fraunhofer-like in character.

[7 marks]

1.8) Explain briefly whether the following statements are true or false.

a) The intensity in the Fraunhofer diffraction pattern of a transmitting object is directly proportional to the Fourier transform of the transmission function that describes the object.

b) The Fraunhofer diffraction pattern from a transparent aperture is identical in all respects to the Fraunhofer diffraction pattern from an opaque mask of the same size and shape.

c) The far-field diffracted intensity from a regular two-dimensional array of identical apertures is the same as that produced when the same apertures are placed at random positions in the object plane.

[7 marks]

## SECTION B – Answer TWO questions

2) State, without proof, the theorem concerning the equivalent surface and volume distribution of polarisation charge for a dielectric body placed in an electric field.

[8 marks]

A parallel plate capacitor is constructed from square metal plates of side $l = 100\,\text{mm}$ separated by a distance $d = 1\,\text{mm}$. The space between the plates is filled with an inhomogeneous dielectric in which the relative permittivity varies linearly from unity at the upper plate to 2.5 at the lower plate.
If $\sigma$ is the charge density on the metal plates show that the electric field strength $E$ at a distance $x$ below the upper plate is

$$E(x) = \frac{\sigma}{\varepsilon_0\,(1 + 1500x)}$$

[4 marks]

Show that the capacitance of this capacitor is 144.9 pF.

[9 marks]

If a potential difference of 100 V is applied between the plates compute the volume density of polarisation charge $\rho_p$ at a point mid-way between the plates.

[9 marks]

3) State Maxwell's differential equations for the electromagnetic field giving the SI units of all of the quantities involved.

[6 marks]

Why is it necessary to introduce the displacement current into electromagnetic theory.

[8 marks]

The electric field of an electromagnetic wave propagating in free space is given as

$$E_x = 0; \qquad E_y = 0; \qquad E_z = E_0 \sin(\omega t - \beta x)$$

where $x$, $y$ and $z$ are cartesian coordinates, $t$ is time, $E_0$ and $\omega$ are constants and $\beta = \omega\left(\mu_0 \varepsilon_0\right)^{1/2}$.
State the equations relating $\beta$ to the velocity and wavelength of the wave. If the wave frequency is 10 MHz calculate its wavelength.

[4 marks]

Use Maxwell's curl $\mathbf{E}$ equation to derive a formula for the magnetic field strength $\mathbf{H}$. Show that the peak amplitude of $\mathbf{H}$ is $H_0 = \beta E_0 / \left(\omega \mu_0\right)$.

[8 marks]

If the r.m.s. amplitude of the electric field is $1\,\mu$V/m, calculate the r.m.s. amplitude of the magnetic field.

[4 marks]

4 (a) Prove that the external work done to produce a potential difference $V$ between the plates of a capacitor of capacitance $C$ is $\frac{1}{2}CV^2$. Use this result to show that the energy density of an electrostatic field is

$$U = \frac{1}{2}\mathbf{D} \cdot \mathbf{E}$$

[15 marks]

(b) A linear diffraction grating has a periodic distance $d = 80\,\mu$m, and each slit in the grating has a width of $a = 20\,\mu$m. Each slit in the grating may be assumed to be very long so it can be treated as a regular one-dimensional array of identical slits. A 2.4 cm wide piece of the grating is illuminated by a parallel beam of green light from a magnesium source. There are three different wavelengths present in the illumination, with values of 516.7 nm, 517.3 nm and 518.3 nm.

(i) Write down an expression for the chromatic resolving power of the grating, defining the symbols used.

[3 marks]

(ii) Determine whether or not it would be possible to resolve the three wavelengths using any of the first five diffraction orders produced by the grating. (Exclude the zero-order, or undiffracted, light from your discussion).

[12 marks]

5 (a) A particular object is illuminated with a monochromatic plane wave of wavelength $\lambda$ and the diffracted intensity in the far field is found to have the form

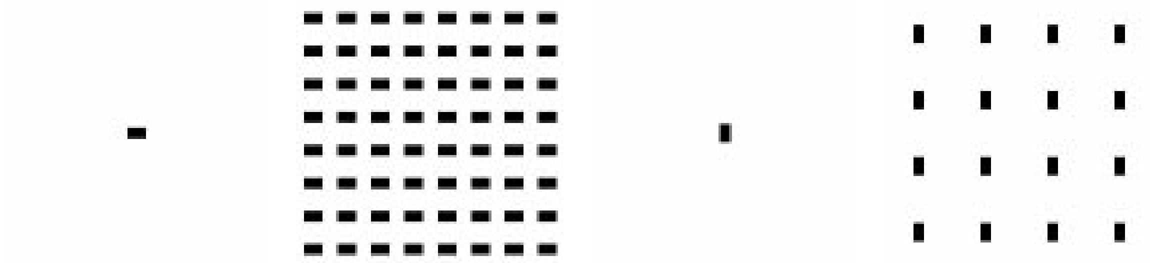
$$I(u,v) = I_0 \left( \frac{J_1\left(\pi a \sqrt{u^2+v^2}\right)}{\pi a \sqrt{u^2+v^2}} \right)^2 \left( \frac{\sin 3\pi d_1 u}{\sin \pi d_1 u} \right)^2 \left( \frac{\sin 5\pi d_2 v}{\sin \pi d_2 v} \right)^2$$

where $J_1()$ represents a first-order Bessel function of the first kind.
Without carrying out a detailed calculation, explain clearly the deductions you can make about the general form of the diffracting object.

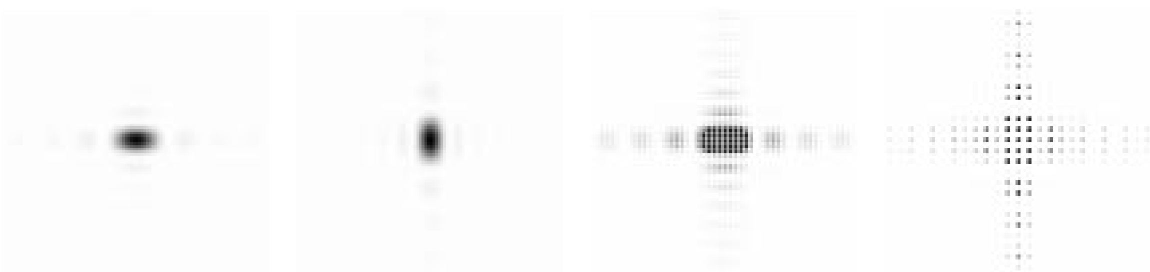
[14 marks]

(b) Figure 5.i shows four objects from which the Fraunhofer diffraction pattern is to be obtained. Figure 5.ii shows four possible Fraunhofer diffraction patterns. Identify which (if any) of the diffraction patterns could be associated with each of the diffracting objects shown, briefly stating a reason for each of your choices.

[16 marks]



5.i Object 1 5.i Object 2 5.i Object 3 5.i Object 4



5.ii Diffraction pattern A 5.ii Diffraction pattern B 5.ii Diffraction pattern C 5.ii Diffraction pattern D

*Note:* In these figures, higher intensities correspond to darker tones, low values appear white.