King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/2470 Principles of Thermal Physics

January 2005

Time allowed: THREE Hours

**Candidates should answer ALL parts of SECTION A,
and no more than TWO questions from SECTION B.
No credit will be given for answering further questions.**

The approximate mark for each part of a question is indicated in square brackets.

**You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.**

TURN OVER WHEN INSTRUCTED

Physical Constants

Permittivity of free space	ϵ_0	=	8.854×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
Speed of light in free space	c	=	2.998×10^{8}	$\mathrm{m\,s^{-1}}$
Gravitational constant	G	=	6.673×10^{-11}	$\mathrm{N\,m^2\,kg^{-2}}$
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_e	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_u	=	1.661×10^{-27}	kg
Proton rest mass	m_p	=	1.673×10^{-27}	kg
Neutron rest mass	m_n	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	$\mathrm{J\,s}$
Boltzmann constant	k_B	=	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ	=	5.670×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
Gas constant	R	=	8.314	$\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_A	=	6.022×10^{23}	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP		=	2.241×10^{-2}	$\mathrm{m^3}$
One standard atmosphere	P_0	=	1.013×10^{5}	$\mathrm{N\,m^{-2}}$

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Throughout this paper, T denotes the temperature, V the volume and P the pressure. C_P and C_V respectively denote the heat capacity at constant pressure and the heat capacity at constant volume. $\gamma = C_P/C_V$ and n is the number of moles.

SECTION A – Answer ALL parts of this section

1.1) Boiling water, at the pressure $P = 1$ atm, is poured into twice the same quantity of liquid water at 0 ^{o}C. Explain how to obtain the temperature of the mixture and give its value in degrees Celsius (the heat capacity of water may be supposed independent of the temperature)

[7 marks]

1.2) Describe the different phases of a 4-stroke engine and explain in which one work is given to the surroundings.

[7 marks]

1.3) Define the Gibbs free energy G and derive an expression for its differential dG in terms of T, P, V and the entropy S. Hence deduce the associated Maxwell relation.

[7 marks]

1.4) An engine A operates in a reversible cycle and is successively in contact with two heat sources Σ_1 and Σ_2 at the temperatures T_1 and T_2 such that $T_1 > T_2$. The global system (A, Σ_1, Σ_2) is isolated. Define the efficiency η of the engine and, using the second law, show that $\eta = 1 - T_2/T_1$.

[7 marks]

1.5) A gas has entropy S and exchanges the heat δQ with the surroundings during an infinitesimal process at temperature T. Explain why and in which circumstances the change in entropy is given by $dS = \delta Q/T$.

[7 marks]

1.6) Sketch the phase diagram (P, T) of a pure substance, showing the three phases (*solid, liquid, vapour*), and explain how it is possible to go from the liquid state to the vapour state without experiencing any phase transition.

[7 marks]

SECTION B – Answer TWO questions

2) One mole of an ideal gas is involved in the following two-stage process. The gas is first compressed in a reversible and isothermic way at temperature T_0, from the pressure P_0 to $10P_0$. It is then expanded in a reversible and adiabatic way, back to the pressure P_0.

a) Give an expression for the temperature T_1 after this two-stage process. Give the numerical value for T_1, using $\gamma = 1.41$ and $T_0 = 273K$.

[5 marks]

b) Show that the temperature T_k after k similar two-stage processes done in a row is

$$T_k = T_0 \times 10^{\frac{k(1-\gamma)}{\gamma}},$$

and give the numerical value for T_5.

[5 marks]

c) Using the Mayer relation and the definition of γ, show that $C_V = R/(\gamma - 1)$.

[5 marks]

d) Explain why the change in internal energy for the k-th two-stage process is $\Delta U_k = C_V(T_k - T_{k-1})$.

[5 marks]

e) Show that

$$\Delta U_k = \frac{RT_0}{\gamma - 1} \times 10^{\frac{k(1-\gamma)}{\gamma}} \left[1 - 10^{\frac{\gamma-1}{\gamma}}\right].$$

[5 marks]

f) Give an expression for the work exchanged with the surroundings during the k-th two-stage process.

[5 marks]

3) An ideal gas, initial state (P_A, V_A), operates in a reversible cycle that can each be decomposed into three steps. The first step $A \to B$ is an isochoric process where the pressure reaches $P_B > P_A$. The second step $B \to C$ is an adiabatic process which leads to the volume $V_C > V_A$ and the last step is an isobaric process which leads back to the initial state A.

a) Plot the cycle in a Clapeyron diagram (P versus V) and give the signs of the heats Q_{AB} and Q_{CA} exchanged with the surroundings.

[6 marks]

b) Define the efficiency η of the engine and show that it is given by

$$\eta = 1 + \frac{Q_{CA}}{Q_{AB}}.$$

[6 marks]

c) Express Q_{AB} and Q_{CA} in terms of the temperatures T_A, T_B, T_C and the heat capacities C_V and C_P.

[6 marks]

d) Show that the efficiency is given by

$$\eta = 1 + \gamma \frac{a-1}{a} \frac{P_A}{P_B - P_A},$$

where $a = V_A/V_C$.

[6 marks]

e) Using an equation characterizing the adiabatic process $B \to C$, give an expression for η in terms of γ and a only.

[6 marks]

4) The liquid and vapour phases of a pure substance are in equilibrium at temperature T.

a) Explain the concept of saturated vapour pressure P_S and define the quantity L given by the Clausius-Clapeyron equation

$$L = T(v_g - v_l)\frac{dP_S}{dT},$$

where v_g and v_l are the volumes per unit mass of the vapour and liquid phases respectively.

[5 marks]

b) Experiments show that $L = A - BT$ is a good approximation in the range of temperatures considered here, where A and B are constants. The vapour is supposed to be an ideal gas. Neglecting the volume per unit mole of the liquid compared with that of the vapour, show that

$$P_S = P_0 \left(\frac{T_0}{T}\right)^{\frac{BM}{R}} \exp\left\{\frac{AM}{R}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right\},$$

where M is the mass per unit mole and (T_0, P_0) is a reference point.

[8 marks]

c) For water $A = 3.34 \times 10^6$ J kg^{-1} and $B = 2.93 \times 10^3$ J kg^{-1} K^{-1}. If boiling water at atmospheric pressure is taken as the reference point, compute P_S for $T = 433$ K.

[5 marks]

d) From the relation found in b), compute the slope dP_S/dT at 100 ^{0}C.

[4 marks]

e) At 100 ^{0}C, this slope is actually 0.036 atm K^{-1} and $L = 2.256 \times 10^6$ J kg^{-1}. Considering that $v_l << v_g$, compute the volume per unit mass of the saturated vapour at this temperature (reminder: 1 atm$\simeq 10^5$ Pa).

[4 marks]

f) The mass per unit volume of liquid water at 100 ^{0}C is 958 g l^{-1}. Give an estimate, as a percentage, of the error made in the above calculation of v_g by neglecting v_l.

[4 marks]

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