King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/2470 Principles of Thermal Physics

January 2004

Time allowed: THREE Hours

Candidates should answer ALL parts of SECTION A, and no more than TWO questions from SECTION B. No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper. Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Physical Constants

Permittivity of free space	ϵ_0	=	8.854×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
Speed of light in free space	c	=	2.998×10^{8}	$\mathrm{m\,s^{-1}}$
Gravitational constant	G	=	6.673×10^{-11}	$\mathrm{N\,m^2\,kg^{-2}}$
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_e	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_u	=	1.661×10^{-27}	kg
Proton rest mass	m_p	=	1.673×10^{-27}	kg
Neutron rest mass	m_n	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	J s
Boltzmann constant	k_B	=	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ	=	5.670×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
Gas constant	R	=	8.314	$\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_A	=	6.022×10^{23}	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP		=	2.241×10^{-2}	$\mathrm{m^3}$
One standard atmosphere	P_0	=	1.013×10^{5}	$\mathrm{N\,m^{-2}}$

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Throughout this paper, T denotes the temperature, V the volume and P the pressure. C_P and C_V respectively denote the heat capacity at constant pressure and the heat capacity at constant volume. n is the number of moles.

SECTION A – Answer ALL parts of this section

1.1) A mass of 10 kg, which is released from a height of 10 m, falls freely under gravity and hits a piece of iron of mass 1 kg, coming to rest. The heat capacity of iron is 0.5 $\mathrm{J.g^{-1}.K^{-1}}$ and it can be assumed that the temperature of the falling mass does not change. If the acceleration due to gravity is $g \simeq 10\ \mathrm{m.s^{-2}}$, what is the increase in temperature of the iron?

[7 marks]

1.2) A system goes from a state A to a state B in an adiabatic process. Explain why the work exchanged with the surroundings is independent of the thermodynamic path of the adiabatic process.

[7 marks]

1.3) The internal energy of n moles of a diatomic ideal gas is $U = (5/2)nRT$. Using the Mayer relation for the difference of heat capacities, compute the ratio $\gamma = C_P/C_V$ of the heat capacities for this gas.

[7 marks]

1.4) The Gibbs free energy of a gas is given by

$$G(P,T) = C_P T + nRT \ln\left(\frac{P}{P_0}\right) + nRT \frac{C_P}{C_V - C_P} \ln\left(\frac{T}{T_0}\right),$$

where P_0, T_0 are constants and C_P, C_V are the heat capacities at constant pressure and constant volume respectively. Given that $dG = VdP - SdT$ and by considering the partial derivatives of G with respect to its variables, derive an equation of state of the gas.

[7 marks]

1.5) Using the Carnot-Clausius inequality and the First Law applied to cyclic transformations, show that a thermal engine cannot operate with one heat reservoir only. (This is the Kelvin statement of the Second Law)

[7 marks]

1.6) The infinitesimal change in internal energy for one mole of a Van der Waals gas is

$$dU = C_V dT + \frac{a}{V^2} dV,$$

where a is a constant and C_V could be a function of T and V. Using the appropriate Maxwell relation, show that C_V actually depends on the temperature only.

[7 marks]

1.7) $G(P, T, n)$ is the Gibbs free energy of n moles of a substance at temperature T and pressure P. Explain why $G = n\mu(P, T)$, where μ is the chemical potential.

[7 marks]

1.8) A and B are two systems which can exchange heat only and are isolated from the rest of the Universe. Their temperatures are such that $T_A > T_B$. From the relation $(\partial S/\partial U)_V = 1/T$, where S is the entropy and U the internal energy, use the Second Law to show that when A and B are in contact, the heat goes from A to B.

[7 marks]

SECTION B – Answer TWO questions

2) A heat engine operates cyclically, using an ideal gas as the working fluid. Each cycle may be split into 4 steps. The first step (from state A to state B) is an isobaric compression with $P = P_A$. The second step (from state B to state C) is an adiabatic compression until the pressure reaches the value P_C. The third step (from state C to state D) is an isobaric expansion with $P = P_C$ and the last step (from state D back to state A) is an adiabatic expansion where the pressure goes back to its initial value P_A.

a) Draw the cycle in a Clapeyron diagram (P, V) and give the signs of Q_{AB} and Q_{CD}, the heat transfers into the gas during the two isobar processes.

[8 marks]

b) C_P is the heat capacity at constant pressure of the gas. Express Q_{AB} and Q_{CD} in terms of C_P and the temperatures T_A, T_B, T_C and T_D.

[4 marks]

c) From the general definition of the efficiency η and the First Law, show that

$$\eta = 1 + \frac{T_B - T_A}{T_D - T_C}.$$

[5 marks]

d) A characteristic quantity for this engine is $a = P_C/P_A$. Using the equation of state of the gas, derive an expression for η as a function of a and the volumes V_A, V_B, V_C and V_D.

[5 marks]

e) Use the equations of the two adiabatic curves to show that

$$\eta = 1 - a^{\frac{1}{\gamma} - 1},$$

where γ is the ratio C_P/C_V.

[8 marks]

3) A cylinder with adiabatic walls is closed by a piston (mass m, area A) which can move up and down without friction. The cylinder contains an ideal gas. At equilibrium (no motion of the piston) the pressure inside the cylinder is P_1 and the piston is at the height h. The pressure outside the cylinder is P_0 and g is the acceleration due to gravity.

a) Show that $P_1 - P_0 = mg/A$.

[6 marks]

b) When the piston moves in a reversible way from its equilibrium position with displacement z, the pressure inside the cylinder becomes P. Show that

$$P = P_1 \left(\frac{h}{h+z} \right)^{\gamma},$$

where γ is the ratio C_P/C_V.

[8 marks]

c) The deviation from the equlibrium is small ($z << h$), such that

$$\left(\frac{h}{h+z} \right)^{\gamma} \simeq 1 - \gamma \frac{z}{h}.$$

Write the equation of motion for the piston and show that this motion is a harmonic oscillation with angular frequency

$$\omega = \sqrt{\gamma \frac{P_1 A}{mh}}.$$

[10 marks]

d) Show that the dependence on temperature is such that ω is proportional to $1/\sqrt{T}$.

[6 marks]

4) The vapour and liquid states of a pure substance are in equilibrium at temperature T. The mass fraction of the substance which is vapour is x. The heat capacities per unit mass are c_v and c_l for the vapour and the liquid phases, respectively, and the volumes per unit mass are u_v and u_l. L is the latent heat (defined per unit mass) at temperature T. The processes which occur in this system are reversible. q and s are the heat and entropy per unit mass, respectively.

a) Show that, when the temperature changes by dT, the heat per unit mass exchanged with the surroundings is

$$\delta q = x c_v dT + (1-x) c_l dT + L dx.$$

[8 marks]

b) Write down, in terms of dT and dx, the associated change of entropy per unit mass ds, and show that the corresponding Maxwell relation gives

$$c_v - c_l = \frac{dL}{dT} - \frac{L}{T}.$$

[8 marks]

c) Using the previous equality, show that

$$ds = d\left(\frac{xL}{T}\right) + c_l \frac{dT}{T}.$$

[8 marks]

d) Hence show that the equation corresponding to the adiabatic processes occuring in this system is (c_l is considered constant):

$$\frac{xL}{T} + c_l \ln\left(\frac{T}{T_0}\right) = \frac{xL_0}{T_0},$$

where L_0 is the latent heat at a given temperature T_0.

[6 marks]

5) A drop of liquid has surface area A and surface tension ψ in the air, i.e. the work necessary to increase the area by dA is $\delta W = \psi dA$.

a) Show that the differential of the internal energy of the drop is $dU = TdS + \psi dA$, where S is the entropy of the drop.

[4 marks]

b) Considering the Helmholtz free energy $F = U - TS$, show that

$$\left(\frac{\partial S}{\partial A}\right)_T = -\left(\frac{\partial \psi}{\partial T}\right)_A .$$

[5 marks]

c) The heat capacity at constant area C_A is defined by $dU = C_A dT + (f + \psi)dA$, where f is a function which can depend on T and A. Comparing the above expression for dU and that given in part a), show that

$$f = -T\left(\frac{\partial \psi}{\partial T}\right)_A .$$

[8 marks]

d) The surface tension is given by $\psi = aT + b$, where a and b are constants. Determine a corresponding expression for f.

[4 marks]

e) Using the Maxwell relation corresponding to the expression dU given in question c), show that the heat capacity C_A is independent of area A.

[4 marks]

f) The drop is spherical and $b > 0$. Determine the sign of the change in internal energy if we split the drop into two identical spherical drops at constant temperature. Is the drop likely to split itself spontaneously?

[5 marks]