1 STRUCTURE OF THE ATOM

1.1 DISCOVERY OF THE ELECTRON: THOMSON'S EXPERIMENT

In 1897, J.J. Thomson designed an experiment to measure the charge to mass ratio e/m of the particles in a Cathode Ray Tube. The beam of cathode rays was accelerated by the applied PE between the cathode and anode; a transverse electric field was then applied to the beam as it passes through a pair of deflecting plates.

$$E = \frac{V}{d} = \frac{\text{Voltage drop between plates}}{\text{Distance between plates}} \,. \tag{1}$$

The displacement as measured on the screen is

$$y_2 = \left(\frac{e}{m}\right)\frac{V}{d}\frac{l}{v_0^2}\left(\frac{1}{2}l + L\right) \tag{2}$$

l is length of the plates, L distance from end of plates to the screen and v_0 is the velocity of the electrons. So, measuring V and y_2 we can determine e/m since:

$$y_2 = \text{const}\frac{e}{m}V \tag{3}$$

This gave Thomson the value for the charge to mass ratio, e/m, much larger—by a factor of 1000—from the value expected, given known masses of atoms. This indicated that the particles in the cathode rays were very light. ———$e/m = 1.7588 \times 10^{11}$ C/Kg———. When the experiment was run in reverse, much heavier particles (ions) were found.

1.2 MILLIKAN'S OIL DROP EXPERIMENT

Millikan (1909) measured the rate of fall of droplets of oil through air in the presence of an electric field. Droplets are sprayed through the nozzle into the space between the plates of a condenser. The droplet has charge $q = ne$ where n is an integer and e the basic unit of charge. We have:

$$F = Mg - qE - \beta v \tag{4}$$

where the last term corresponds to the force due to viscosity of the air when moving with speed v and $\beta = 6\pi\eta r$ (η = viscosity and r - radius). First we set $E = 0$. The drop falls and it reaches terminal velocity when:

$$Mg = \beta v_1 \tag{5}$$

since gravitational acceleration is balanced by viscous drag. Now, turn on the electric field: the drop moves upwards. The terminal velocity in this case is reached when:

$$qE = Mg + \beta v_2 = \beta(v_1 + v_2) \ . \tag{6}$$

To calculate β we need to know r. This can be obtained as a function of v_1 using equation 5 and the effective mass: $M = \frac{4}{3}\pi r^3(\rho_O - \rho_A)$

$$r = \sqrt{\frac{9\eta v_1}{2g\left(\rho_O - \rho_A\right)}} \ . \tag{7}$$

ρ_O and ρ_A are the densities of oil and air respectively and η is the coefficient of viscosity. The charge q was found to change by integral units of a basic charge e which was found to be $e = 1.6 \times 10^{-19}$ C. Combining with Thomson's value for e/m the mass of the electron is $m = 9.10 \times 10^{-31}$ kg (1836 $\simeq$ times lighter than a positive hydrogen ion).

1.3 COLLISIONAL CROSS-SECTIONS

1.3.1 TOTAL COLLISIONAL CROSS-SECTION

DEFINITION: The Total Collisional Cross-Section, σ_T, is the effective area, normal to the direction of incidence, provided by a target to an incoming projectile. It is a measure of the probability of an scattering event occuring.

N.B. 'effective' is the key word here. For a hard disk of radius R the effective area= actual area = πR^2. But long-range electrostatic forces mean that atoms interact when they are relatively far away from each other.

Consider a beam of particles of type 'A' and intensity=I where

I=number of particles/unit area/unit time.

The beam is incident on a slab of material containing target particles of type 'B'. The density of target particles, that is, the number of particles per unit volume, is n. Remember can use ideal gas law to find n.

The fraction of incident particles scattered or lost from the beam must be proportional to the number of target particles:

$$dI/I = -n\sigma_T dx \ . \qquad (8)$$

Integrating and setting initial intensity equal to I_0 we get the Beer-Lambert law:

$$I = I_0 \exp(-nl\sigma_T) \ . \qquad (9)$$

σ_T MUST have units of area.

The unit of σ is the BARN: 1 barn $= 10^{-28}\,\mathrm{m}^2$ but is never used in atomic and molecular physics because typical magnitudes for atomic σ_T are $\sim 10^{-20}\,\mathrm{m}^2$.

σ_T is the effective area presented by each atom, B, to the incoming projectile, A. Its values depend on the projectile and its velocity as well as the target.

1.3.2 e^-, e^+ and Ps scattering from He and Ar atoms

The figure 1.2 shows measurements of σ_T for:

e^- + He	e^+ + He	Ps + He
e^- + Ar	e^+ + Ar	Ps + Ar

Some of the interactions between projectile and targets are:

- **static:** interaction with undisturbed target. The projectile sees a partially screened nucleus.

For e^- interaction is attractive (negative)
For e^+ interaction is repulsive (positive)

- **polarization:** electron charge cloud is distorted by approaching projectile.

  Interaction is attractive in both cases

So, at **at low E**, the interactions are additive for the e^- but they approximately cancel for $e^+ \longrightarrow \sigma_T(\text{Ar}) > \sigma_T(\text{He})$

But **at high E**, $\sigma_T(e^-) \simeq \sigma_T(e^+)$. This is because the importance of polarization decreases (it takes time to distort the cloud charge). The magnitude of the static interaction is the same for both projectiles.

1.3.3 INELASTIC CROSS-SECTIONS

Atoms also have internal structure. Their electronic energies are quantised into energy levels E_1, E_2,... (energy spectrum). If an atom is in its ground electronic state and the collisional energy $E > E_2 - E_1$ then the collision may result in an exchange of kinetic and internal energy. The electron may be excited into a higher lying state.

Hence we define also:

- a total inelastic cross-section σ_{ij} from quantum level i to level j;

The sum of all elastic and inelastic cross-sections gives the total cross section.

1.4 RUTHERFORD SCATTERING

Geiger, Marsden and Rutherford carried out experiments on the scattering of α-particles by thin metallic foil.

Most of the α-particles were only weakly deflected ($\theta < 1°$) but remarkably a few scattered by $\theta > 90°$ (i.e., back-scattered).

Concluded that most of the atom's mass and positive charge is concentrated over a small region $r \sim 10^{-14}$ m—THE NUCLEUS—whereas for the atom, $r \sim 10^{-10}$ m.

1.5 DIFFERENTIAL CROSS-SECTION (see fig 1.1).

DEFINITION: The Differential Cross-Section, $\frac{d\sigma}{d\Omega}$, is the particle flux scattered, by each target nucleus, into solid angle $d\Omega$ divided by the incoming intensity.

N.B. flux means particles per unit time
intensity means particles per unit time per unit area.

What Rutherford found was:

$$dN \propto Nnld\Omega \ . \tag{10}$$

where N is the incident flux, n and l are the target density and thickness respectively and $d\Omega$ is the solid angle(see fig 1.1): Introducing a constant of proportionality:

$$dN = \frac{d\sigma}{d\Omega} Nnld\Omega \ . \tag{11}$$

1.6 RELATION BETWEEN TOTAL AND DIFFERENTIAL CROSS-SECTION

If we count the scattered particles over all possible directions we get back the total cross-section.

$$\sigma_T = \int \frac{d\sigma}{d\Omega}(\theta, \phi) d\Omega = \int_0^{2\pi} d\phi \int_0^{\pi} \frac{d\sigma}{d\Omega}(\theta, \phi) \sin\theta d\theta \ . \tag{12}$$

(N.B. $\int d\Omega = 4\pi$.)

There is an important simplification for the case of inter-atomic collisions. In this case, since we have cylindrical symmetry, $d\sigma/d\Omega$ does not depend on ϕ.

So we have:

$$\sigma = 2\pi \int_0^{\pi} \frac{d\sigma}{d\Omega}(\theta) \sin\theta d\theta \ . \tag{13}$$

See figures 1.3 and 1.4 for examples.

1.7 QUANTUM CROSS-SECTIONS

A proper quantum treatment must allow for the wave nature of atoms. Use a time-independent treatment where we replace the incoming beam of particles by plane waves. The scattered particles are described as outgoing spherical waves. These solutions are obtained from the time-independent Schrödinger equation:

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + \ V(r)\right]\psi(\underline{r}) = E\psi(\underline{r}) \ . \tag{14}$$

The potential is 'central' (depends only on $r = |\underline{r}|$ not on $\underline{r}$) for collisions between atoms or between electrons and atoms.

We look for solutions which, as $r \Rightarrow \infty$, take the form:

$$\psi(\underline{r}) = \underbrace{e^{i\underline{k}.\underline{r}}}_{\text{plane wave}} + \underbrace{f(\theta,\phi)\frac{e^{ikr}}{r}}_{\text{spherical waves}} \ . \tag{15}$$

The wavenumber $k = \sqrt{\frac{2mE}{\hbar^2}}$, E is the energy of the projectile.

$f(\theta, \phi)$ is the SCATTERING AMPLITUDE.

$$|f(\theta,\phi)|^2 = \frac{d\sigma}{d\Omega}(\theta,\phi) \qquad \text{and} \qquad \sigma_T = \int |f(\theta,\phi)|^2 d\Omega.$$

Solution different from the hydrogen atom.

- $E > 0$.
- Hence we do not have bound state solutions,
  i.e., $\psi \neq 0$ for $r \Rightarrow \infty$.

- So we don't get discrete eigenvalues E_n but a continuously varying eigenvalue E.

For inelastic quantum scattering we would look for solutions for which the outgoing waves have different energy (different wave-number) from the incoming waves, i.e.,

$$\psi(\underline{r}) = \underbrace{e^{ik_i z}}_{\text{plane wave}} + \underbrace{f_{ij}(\theta,\phi)\frac{e^{ik_f r}}{r}}_{\text{spherical waves}} \quad . \tag{16}$$