Atomic and Molecular Physics, Solutions to Problem Sheet PHAS2224.1
Issued Thursday 24 January 2008, due Thursday 31 January 2008

1. In Millikan's experiment oil droplets were sprayed through a nozzle into the space between a pair of condenser plates. **[1]**
   Friction at the nozzle removes electrons leaving the droplets positively charged. **[1]**
   Millkan measured the terminal velocity of the droples when falling freely, then applied a potential difference to the condenser plates to reverse the direction of the droplets and measured the upwards terminal velocity (which depends on the charge carried). **[1]** **[1]**
   The charge on the droplet could be changed using a burst of ionising radiation, and the experiment repeated. **[1]**
   If the viscous force on the droplet is known then the charge carried by the droplet can be determined. This was found to occur in integer multiples of an amount $e = 1.602 \times 10^{-19}$ C, providing evidence that electric charge is quantised, and that e is a fundamental unit of electric charge. **[1]**

   In the zero electric field case the forces acting on the drop are:

   $$F = M'g - \beta v_1$$

   where M' is an effective mass that accounts for the buoyancy of the drop in air, i.e.

   $$M' = M_{\text{oil}} - M_{\text{air}} = \frac{4}{3}\pi r^3(\rho_{\text{oil}} - \rho_{\text{air}}),$$

   and $\beta = 6\pi\eta r$ is the Stokes drag coefficient.

   At terminal velocity the net force is zero and so

   $$M'g = \beta v_1$$

   Inserting expressions for M' and β and rearranging we have for the radius:

   $$r = \sqrt{\frac{9\eta v_1}{2g(\rho_{\text{oil}} - \rho_{\text{air}})}}$$

   which from the data in the question gives us:

   $$r = \sqrt{\frac{9 \times 1.8 \times 10^{-5} \times 4.35 \times 10^{-4}}{2 \times 9.81 \times (900 - 1.29)}} = 2 \times 10^{-6} \text{ m}.$$

   When the electric field $E = V/d$ is applied at the new terminal velocity v_2 we have

   $$q\frac{V}{d} = M'g + \beta v_2$$

   where we can now substitute for $M'g$ to get

   $$q\frac{V}{d} = \beta(v_1 + v_2)$$

And so:

$$q = \frac{d}{V} \times 6\pi\eta r(v_1 + v_2)$$

Inserting values:

$$q = \frac{5 \times 10^{-3}}{3000} \times 6\pi \times 1.8 \times 10^{-5} \times 2 \times 10^{-6} \times (4.35 + 1.38) \times 10^{-4}$$

or

$$\underline{q = 6.48 \times 10^{-19}\ \mathrm{C} = 4.05e}$$

where e is the fundamental charge of the electron. **[4]**

2. In the Rutherford experiment the deflection of alpha particles scattered by a thin gold foil was observed. Most alpha particles pass straight through or deviate only by a small amount (1°). A few are scattered by large angles, and some are even backscattered (> 90°). **[3]**

From these results it was deduced that much of the atom was 'empty space', and that the positive charge and almost all the mass is concentrated in a small nucleus around which the electrons orbit. The scattering can then be described by the dynamics of a charged particle in a Coulomb field leading to the famous Rutherford $\csc^4$ law. **[3]**

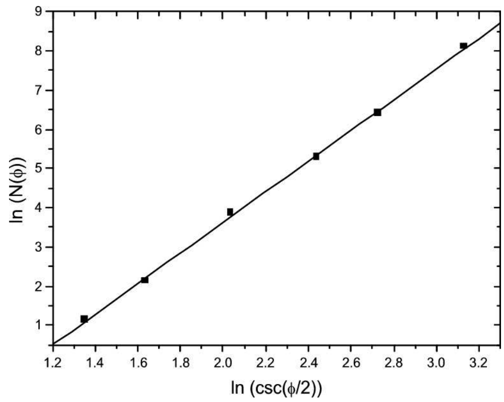

Figure 1: The required graph of $\ln(N(\phi))$ *vs* $\ln(\csc(\phi/2))$

To verify that the data fits the Rutherford scattering law, plot a graph of $\ln(N(\phi))$ *vs* $\ln(\csc(\phi/2))$. The graph should be a straight line, as shown in the figure above.

The gradient of the fitted straight line is 3.9 (approx. 4) **[4]**

3. Bohr's postulates are:

   (a) The electron moves in certain allowed circular classical orbits (stationary states) about the nucleus without radiating

   (b) Emission or absorption of radiation by an atom is associated with a transition between these states **[2]**

Balancing the forces acting on the electron in a circular orbit, we find:

$$\underbrace{\frac{Ze^2}{4\pi\epsilon_0 r^2}}_{\text{Coulomb}} = \underbrace{\frac{m_e v^2}{r}}_{\text{Centripetal}} .$$

Imposing the quantisation of angular momentum $m_e r v = n\hbar$, with n an integer, gives:

$$\frac{Ze^2}{4\pi\epsilon_0 r^2} = \frac{m_e}{r}\left(\frac{n\hbar}{m_e r}\right)^2 .$$

Rearranging gives us the allowed radii, r_n:

$$r_n = \frac{4\pi\epsilon_0 \hbar^2 n^2}{Ze^2 m_e}$$

Inserting this into the equation defining the quantisation of angular momentum gives:

$$v_n = \frac{Ze^2}{4\pi\epsilon_0 n\hbar},$$

And so for the total energy, $E = T + V$:

$$E = \underbrace{\frac{1}{2}m_e v_n^2}_{\text{kinetic}} - \underbrace{\frac{Ze^2}{4\pi\epsilon_0 r_n}}_{\text{potential}}$$

Inserting derived expressions for r_n and v_n we get:

$$E_n = \frac{1}{2}m_e\left(\frac{Ze^2}{4\pi\epsilon_0 n\hbar}\right)^2 - \frac{Ze^2}{4\pi\epsilon_0}\times\frac{Ze^2}{4\pi\epsilon_0}\left(\frac{\hbar n}{m_e}\right)^2 m_e = -\underbrace{\frac{1}{2}m_e\left(\frac{e^2}{4\pi\epsilon_0\hbar}\right)^2}_{\text{constants}} Z^2\frac{1}{n^2}$$

We can group the constants together and call them the Rydberg constant:

$$R = \frac{1}{2}m_e\left(\frac{e^2}{4\pi\epsilon_0\hbar}\right)^2,$$

giving:

$$\underline{E_n = -R\frac{Z^2}{n^2}}$$

[4]

The difference between the Rydberg constants for hydrogen (H) and ionized helium (He^+) may be accounted for by the different *reduced masses* of the two atoms. In accounting for the finite nuclear mass in the Bohr model we can replace the mass of the electron, m_e, with the reduced mass of the atoms $\mu = M_N m_e/(M_N + m_e)$ with M_N the nuclear mass. Since the nuclei have different masses we expect the corresponding Rydberg constants to be different. **[2]**

To completely remove the electron requires:

$$E = Z^2 R_{\mathrm{He}^+} \times hc = 4 \times 109722 \times (6.626 \times 10^{-34}) \times (2.998 \times 10^{10}) = \underline{8.72 \times 10^{-18}\ \mathrm{J}}$$

Alternatively, $\underline{E = 54.4\ \mathrm{eV}}$. **[2]**