Diploma in Astronomy—DP11 First Term 2005-6

Solar System: Model Solutions for Problem Paper 4.
15 December 2005

Solutions were due by Tuesday 13 December. After that date they could be accepted, but once these model solutions are posted, they cannot receive credit for marks (but can count towards completion of the course).

1. Imagine you were part of a team of engineers in charge of landing an astronaut on Venus. The astronaut would have to be able to descend through the atmosphere and then move around on the surface taking samples of rocks, atmosphere, etc. Survival for 24 hours is required, followed by ascent back to an orbiting spacecraft. Your job is to create an environment suit. (Someone else is designing the lander.) List, in a little essay or paragraph, the specific hazardous conditions that exist in the atmosphere or on the surface, and outline what means the environment suit might use to provide protection against them. [200-300 words] (8)

Answer: Some latitude is allowed in the answers to this essay question, but for full marks the answer should mention the following Venusian hazards: protection against high temperature (may require thick asbestos lining and/or nuclear-powered air conditioner), protection against high pressure (may require armour similar to extreme deep-sea suit), breathable air supply and sealant against CO_2 leakage. Other hazards include protection against hot sulfuric acid droplets in the clouds during descent and ascent (i.e, in the atmosphere). There is a penalty (subjectively assessed according to lecturer-annoyance factor) for going substantially over the 300 word limit. A complete answer under the 200-word limit is not penalised.

Award up to 1.5 marks for each principal hazard (T, P, air) and up to 1 mark for all additional requirements (acid protection, food, water). Award up to 2 marks for lack of errors in English composition. However, maximum is 8 marks so good English can offset problems elsewhere.

2. The Galilean Moons are said by experts to have a "Laplace resonance" in their periods, close to but not exactly the simple ratio of 1:2:4 for Io (satellite 1), Europa (2) and Ganymede (3) as given in the text.

(a) This part is about using the right number of significant figures in a calculation. If the mean motion n is defined as $n = 360°/\text{Period(days)}$ and $P_1 = 1.769138\,\text{d}$, $P_2 = 3.551181\,\text{d}$, and $P_3 = 7.154553\,\text{d}$, verify to an appropriately correct number of significant figures that the Laplace resonance

$$n_1 - 3n_2 + 2n_3 = 0$$

is indeed precise. (5)

(Show your working, don't just say, "It is verified," without demonstrating it with numbers.)

Answer: The information is given to 7 significant figures for each period, so we want our verification to be that good. We recognise that the final digit may be uncertain by ± 0.5 so we should not expect *perfect* agreement. To avoid rounding errors, ideally we should carry out the calculations to 8 significant places and round to 7 in the final result, but if you simply carried 7 significant figures that was accepted. Substitute the periods given in the equation

$$360/P_1 - 3 \times 360/P_2 + 2 \times 360/P_3 = 0$$

to obtain (carrying 8 significant digits)

$$203.48893 - 304.12418 + 100.63522 = -0.00003$$

which is 000.0000 to 7 significant figures. Thus we have verified that the Laplace resonance is precise at this level of accuracy.

(b) By what percentage is the 1:2:4 approximate resonance not accurate? For this part, look at the worst case of the three in which the true ratio is not equal to the approximate one. (4)

Answer: If the 1:2:4 relationship (given in the textbook) was exact, then for example, we would have $(P_2 - 2 \times P_1)/P_2 = 0.000000$ which should be exact. But it isn't: you get 0.00363 or an approximate error of 0.36 %.
There are two other pairs of values to check:
$(P_3 - 2 \times P_2)/P_3 = 0.00729$ or 0.73%;
$(P_3 - 4 \times P_1)/P_3 = 0.01090$ or 1.09%.
The worst case is off by 1.09%.

(c) In Robert A. Heinlein's 1950 science-fiction novel *Farmer in the Sky* about future colonists on Ganymede, the exciting climax of the story occurs when a rare line-up of the Galilean Satellites takes place and all four are in a line on the same side of Jupiter, causing a huge earthquake (or Ganymede-quake) that threatens the survival of the colony. Use information from the lecture on 29th November, and the textbook, and explain why this could never happen. (3)

Answer: You can never get a line-up of all four moons on one side of Jupiter. The textbook explains that whenever Europa and Ganymede are on one side of the planet, Io is always on the other side. So the most you can get is Europa, Ganymede and Callisto all lining up.

Some students gave elaborate answers, but this was all we were seeking.

Total marks possible: 20