Diploma in Astronomy—First Term 2005-6

Solar System: Problem Paper 2. Model Answers

22nd November 2005

Solutions had to be returned by Tuesday 15 November. After that date they could be accepted, but once these model solutions are posted, they cannot receive credit for marks (but can count towards completion of the course). The weighting for each question is given in ().

1. A (fictitious) superior planet, Kreplach, was observed from Earth by the ancient Gladhandians over many centuries. They found an average synodic period of 982.657 days between oppositions. Assuming the sidereal period of Earth is 365.256 days (as given in lectures), calculate the sidereal period of Kreplach. (5)

Answer: All the questions can be answered by reference to the lecture notes now on the web site. Let P_K be the sidereal period of Kreplach, Y be the sidereal period of Earth, and S_K be the synodic period of Kreplach, then

$1/P_K = 1/Y - 1/S_K$

and following the Mars example,

$1/P_K = 1/365.256 - 1/982.657 = 0.001720156725$

and $P_K = 581.342$ days (Earth days, of course).

For full marks the answer must retain all the significant figures of the original data (6 figures for the least precise number), no more, no less. I was a bit lenient on this. Remember, keeping the correct number of significant digits is important.

2. Use the results from (1), plus Kepler's third law, $P^2 = ka^3$, to deduce the semi-major axis a of Kreplach's orbit in astronomical units. Explain your choice of units and state what value of k should be adopted (see lecture notes). How does Kreplach's a compare with Mars' ($a = 1.524$ AU)? (5)

Answer: To use Kepler's third law, convert P to sidereal years and get the result for a in astronomical units (AUs). The value of k in these units is 1. This assumes the central body has one solar mass and the mass of the planet is small by comparison.

$P_K = 527.467/365.256 = 1.59160$ years (retain significant figures)

$a = \sqrt[3]{1.59160^2} = 1.36319$ AU

This is significantly smaller than Mars' orbit, so we expect Kreplach to be warmer than Mars but cooler than Earth. Here, I allow full marks for getting a to 4 significant figures (1.363).

3. In the lectures on Earth we talked about the Greenhouse Effect and the way in which a planet's expected mean temperature can be calculated. Assume that Kreplach is a "terrestrial" planet with a nearly circular orbit. Use the semi-major axis of Kreplach's orbit from (2), assume that its rotation period is 15.5 Earth hours (what does this imply for the temperature calculation?), assume that its Bond Albedo is 0.29, and assume that it has a Greenhouse temperature increase of 27 degrees K due to CO_2 and H_2O in its atmosphere.

Deduce the mean temperature of Kreplach. Comment on whether it is possible for liquid water to exist on the surface. (10)

Hint: Remember that the Solar Constant for Earth, 1367.5 watts/m^2, does not apply to Kreplach; you will need to use the Inverse Square Law to deduce the energy received per square metre at Kreplach's distance from the Sun (see Chapter 19-2). You may scale the value for Earth if you wish, as long as you do this correctly (it's much easier calculation).

Answer: The rapid rotation means that we may assume that the average day and night temperatures are the same for the purpose of calculating the planetary mean temperature, just as in the case of Earth. (By contrast, on a slowly rotating planet the sunward side gets very hot, while the night side gets very cold. Examples: Mercury, the Moon.)

It is simplest to scale the solar constant value for Earth to Kreplach's orbit: the value is

$1367.5(1.00000/1.36319)^2 = 735.89$ watts/m^2 and full credit for this part was given whether this method was used or the solar constant was calculated from $L_{\rm sun}/4\pi a^2$, as long as the result was correct. I allowed the answer to be out by 1-2 W/m^2.

The Bond Albedo = 0.29.

Using the formulation in the lectures, the relevant factor is $(1 - A) = 0.71$:

$T_K = \sqrt[4]{\frac{0.71\times 735.89}{4\times 5.67\times 10^{-8}}} = 219.08$ K

The Greenhouse Effect adds 27 degrees to this, so the mean temperature is $T_K = 246.08$ K = -27 C. Kreplach is a cold planet, mostly frozen, but the equatorial region may possibly get warm enough for a little ice to melt during the days. The exact situation would depend on the atmospheric pressure, not specified in the problem.

Total marks possible for Paper 2: 20