Answer SIX questions from section A and THREE questions from section B.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

SECTION A **[Part marks]**

1. Write down the second order partial differential equation describing the propagation of a wave in one dimension with speed c. **[2]**

   A disturbance $y_1(x,t) = A\cos(kx - \omega t)$ is superposed on another disturbance $y_2(x,t) = A\cos(k'x - \omega' t)$. Derive an expression for the resultant, and sketch it. **[5]**

   You may find it useful to note that $\cos(a) + \cos(b) = 2\cos(\frac{1}{2}(a+b))\cos(\frac{1}{2}(a-b))$.

2. State the changes of phase which occur when light is reflected (a) in air from an air-glass interface, (b) in glass from a glass-air interface. **[2]**

   Two flat glass sheets touch at one edge and are separated by a hair 100 mm from that edge, to form a wedge-shaped air gap. When illuminated with light with a wavelength of 600 nm straight dark fringes are observed in reflection with a spacing of 2 mm. Sketch this arrangement, showing typical rays, and calculate the thickness of the hair. **[5]**

3. Write down an expression for the speed of longitudinal waves in a solid rod of density ρ and Young's modulus Y. **[2]**

   A metal bar $1\ \mathrm{m}$ long with $Y = 150 \times 10^9\ \mathrm{N\ m^{-2}}$ and density 6000 $\mathrm{kg\ m^{-3}}$ is suspended horizontally by a thread at each end and longitudinal elastic waves are excited in it. Sketch the variation of (a) stress and (b) particle velocity along the bar when it is vibrating at its lowest natural frequency, and calculate that frequency. **[5]**

4. Show that the frequency f' measured by an observer who is approaching, at a speed v_o, a stationary source which emits sound at a constant frequency f is

$$f' = f(1 + v_\mathrm{o}/v),$$

   where v is the velocity of sound. State the full expression describing the situation when the source moves towards the observer at a speed v_s *and* the observer approaches the source at a speed v_o. **[5]**

   A stationary siren emits a steady note of 1500 Hz. What frequency will be heard by a driver approaching the siren at 33 $\mathrm{m\ s^{-1}}$ if the speed of sound in air is 330 $\mathrm{m\ s^{-1}}$? **[2]**

5. A thin converging lens could be used to focus the Sun's rays on a sheet of paper with the paper held 50 mm from the lens. Sketch a diagram showing how the same lens may be used to form an image of a candle flame on a sheet of paper. Will the image of the flame be upright or inverted? **[4]**

   Where should the lens be placed if the paper is 200 mm from the candle? **[3]**

6. Coherent light of wavelength λ shines through two narrow parallel slits in a screen. Calculate the difference in path lengths from the two slits to a distant point at an angle θ away from the normal to the screen. Hence show that bright fringes will be observed with a spacing $\lambda D/d$ on a screen at a distance D. **[5]**

   In an experiment, two slits 0.1 mm apart were used to form an interference pattern on a screen at a range of 1 m. The bright fringes were found to be 6 mm apart. What was the wavelength of the light? **[2]**

7. Draw and annotate a diagram of a Michelson interferometer, including a compensating plate, showing clearly the ray paths from the source to the detector. **[5]**

   A Michelson interferometer is set up to give straight fringes with monochromatic light from a cadmium source. The mirror at the end of one arm is moved until 1000 fringes have passed. If the distance moved by the mirror is 0.32 mm, what is the wavelength of the light? **[2]**

8. Explain the difference between *phase velocity* and *group velocity*. **[2]**

   Show that the group velocity $v_{\rm g}$ may be written in terms of the phase velocity $v_{\rm p}$ and the wavevector k as

   $$v_{\rm g} = v_{\rm p} + k\frac{{\rm d}v_{\rm p}}{{\rm d}k}.$$ **[3]**

   The approximate dispersion relation for waves on a thin elastic plate is $\omega = Ak^2$ where A is a constant. Find the relationship between the phase velocity and the group velocity for these waves. **[2]**

SECTION B

9. Derive the equation of motion for compressional waves in a fluid of normal density ρ and adiabatic bulk modulus B, and hence show that the speed of sound in the fluid is $v = \sqrt{B/\rho}$. **[6]**

   Explain what is meant by the specific acoustic impedance Z of a fluid, and derive an expression for this quantity in terms of B and ρ. **[4]**

   The energy density in a compression wave with displacement amplitude a and frequency ω is $\frac{1}{2}\rho\omega^2 a^2$, and the amplitude reflected when a wave of unit amplitude is incident normally in medium 1 characterised by ρ_1, B_1 and Z_1 on an interface with medium 2 characterised by ρ_2, B_2 and Z_2 is $(Z_1 - Z_2) / (Z_1 + Z_2)$. Use the principle of conservation of energy to deduce the magnitude of the amplitude transmission coefficient in terms of Z_1 and Z_2. **[5]**

   For a gas, the adiabatic bulk modulus $B = \gamma P$, where P is the ambient pressure. Calculate the speed of sound in air ($\rho = 1.28$ kg m^{-3}, $\gamma = 7/5$) and in Helium ($\rho = 0.18$ kg m^{-3}, $\gamma = 5/3$) at atmospheric pressure (0.1 MPa). How does the difference explain the fact that divers breathing a mixture containing helium speak in a higher-pitched voice? **[5]**

10. Draw a diagram of a Fabry-Perot interferometer, illustrating the main factors which control its resolving power. **[6]**

A Fabry-Perot etalon is formed from part-silvered glass plates a distance d apart. It is illuminated at normal incidence with a parallel beam of light of wavelength λ. Show, by considering light reflected and transmitted at the inner surfaces of the etalon, how the amplitude of the light transmitted may be expressed in the form of a geometric series. Show that the sum of the infinite series may be written in the form

$$E = E_0 T \frac{1}{1 - Re^{4\pi di/\lambda}},$$

making clear the significance of T and R. **[5]**

Hence show that the intensity of light transmitted may be written as

$$I = I_0 \left(\frac{T}{1-R}\right)^2 \frac{1}{1 + F\sin^2(2\pi d/\lambda)},$$

giving an expression for F. **[5]**

Calculate the ratio of the maximum to minimum transmitted intensity when F is (a) 0.5 (b) 500. **[4]**

11. Explain what is meant by *coherence* of a wave pattern, distinguishing between longitudinal and transverse coherence. **[3]**

Show that the total electric field in the Fraunhofer diffraction pattern produced from monochromatic light with wavelength λ by a grating with N narrow slits at uniform spacing d is given by

$$E = E_1 \frac{1 - e^{2iN\gamma}}{1 - e^{2i\gamma}},$$

where E_1 is the field due to one slit and **[5]**

$$\gamma = \frac{\pi d \sin(\theta)}{\lambda}.$$

Hence show that the intensity pattern may be written as

$$I(\theta) = I(0) \left[\frac{\sin(N\gamma)}{N \sin(\gamma)} \right]^2.$$

[3]

Sketch the resulting pattern, and indicate how it would be modified if the slits in the grating have finite width. **[5]**

A grating ruled with 600 lines/mm is illuminated with light of wavelength 600 nm. How many orders of diffraction will be visible? Calculate the angular positions of the principal maxima in the diffraction pattern. **[4]**

12. Explain, with the aid of diagrams, what are meant by the *principal points*, *principal foci* and *nodal points* of a thick lens system. **[6]**

A compound microscope has an objective with a focal length of 5 mm and an eyepiece with a focal length of 20 mm. Draw a ray diagram of the instrument set up with the final image at infinity. If the distance between the lenses is 220 mm, how far from the objective must the object be placed in this arrangement? **[8]**

What is the magnifying power of the microscope when set up as described? You may take the near point distance to be 250 mm. **[6]**

13. Describe the difference between the conditions under which Fraunhofer and Fresnel diffraction may be observed. Derive an expression for the minimum distance D at which a screen must be placed from a slit of width w in order to observe Fraunhofer diffraction with light of wavelength λ. **[6]**

Show that the intensity distribution in the Fraunhofer pattern of a slit of width w illuminated with light of wavelength λ is

$$I(\theta) = I(0) \left[\frac{\sin(\beta)}{\beta} \right]^2 ,$$

where **[7]**

$$\beta = \frac{\pi w}{\lambda} \sin(\theta).$$

Hence derive the Rayleigh criterion that two slit sources, viewed through an aperture of width w, are resolvable if their angular separation is greater than λ / w. How is this result modified in the case of point sources and a circular aperture? Estimate the angular separation of two stars that could just be resolved at an ultraviolet wavelength of 120 nm by the Hubble Space Telescope, which has an objective mirror of radius 2.4 m. **[7]**

END OF PAPER