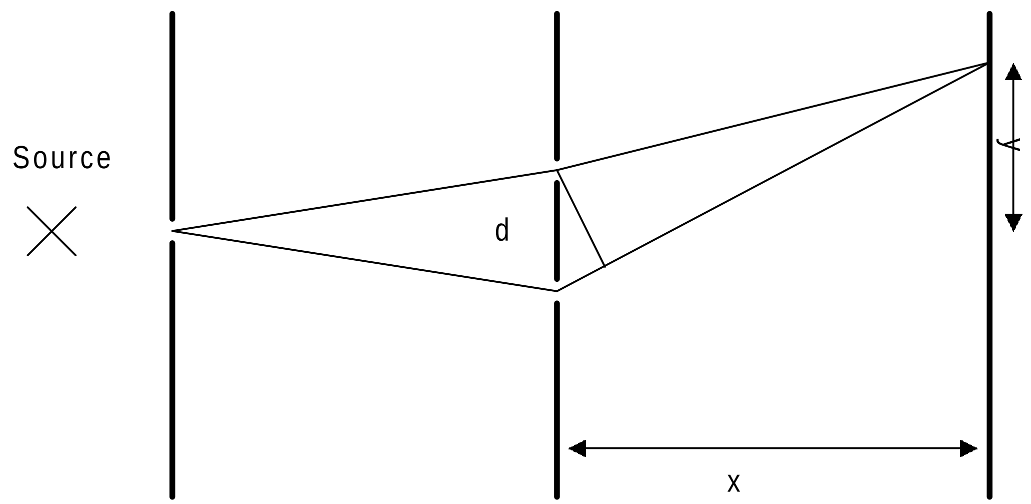
13. The coherence of a wave pattern describes two of its features. Temporal coherence is the extent to which the signals at two times represent two points on the same periodic variation (it relates to the visibility of fringes formed by taking light from one slit and allowing it to interfere after travelling paths of different lengths). Spatial coherence is the extent to which two points at different points on a slice across the beam have a fixed phase relationship to one another (2 marks for one of these, 3 for both)

The expected arrangements two out of Young's slits, the Fresnel biprism, and Lloyd's mirror. (1 mark for one, 3 marks for two)

The figure below shows the geometry of the Young's slits experiment.



The distance from the upper slit to the screen is $d_1 = \sqrt{x^2 + \left(y - \frac{d}{2}\right)^2}$, and that from the lower slit is $d_2 = \sqrt{x^2 + \left(y + \frac{d}{2}\right)^2}$.

Expanding to first order we have, writing $D = \sqrt{x^2 + y^2}$

$d_1 = D - \frac{d \cdot y}{2 \cdot D}$ and $d_2 = D + \frac{d \cdot y}{2 \cdot D}$.

The total amplitude observed on the screen at y will then be

$$A(y,t) = \left(A_1 \cdot e^{\frac{i \cdot k \cdot y \cdot d}{2 \cdot D}} + A_2 \cdot e^{\frac{-i \cdot k \cdot y \cdot d}{2 \cdot D}}\right) \cdot e^{-i \cdot k \cdot D + i \cdot \omega \cdot t}$$

which may be rewritten as

$$A(y,t) = \left[\left(A_1 + A_2\right) \cdot cos\left(\frac{k \cdot y \cdot d}{2 \cdot D}\right) + i \cdot \left(A_1 - A_2\right) \cdot sin\left(\frac{k \cdot y \cdot d}{2 \cdot D}\right)\right] \cdot e^{-i \cdot k \cdot D + i \cdot \omega \cdot t}$$

leading to an intensity pattern

$$I(y) = \left(A_1 + A_2\right)^2 \cdot \left(cos\left(\frac{k \cdot y \cdot d}{2 \cdot D}\right)\right)^2 + \left(A_1 - A_2\right)^2 \cdot \left(sin\left(\frac{k \cdot y \cdot d}{2 \cdot D}\right)\right)^2$$

Simplifying this further,

$$I(y) = \left[A_1^2 + A_2^2 + 2\cdot A_1\cdot A_2\cdot\left[\left(\cos\left(\frac{k\cdot y\cdot d}{2\cdot D}\right)\right)^2 - \left(\sin\left(\frac{k\cdot y\cdot d}{2\cdot D}\right)\right)^2\right]\right]$$

or

$$I(y) = A_1^2 + A_2^2 + 2\cdot A_1\cdot A_2\cdot\cos\left(\frac{k\cdot y\cdot d}{D}\right).$$

This shows that bright lines will be seen on the screen when the argument of the cosine is an integer multiple of 2π, that is when

$$\frac{k\cdot y\cdot d}{D} = 2\cdot n\cdot\pi,\ \ y = n\cdot\frac{2\cdot\pi\cdot D}{k\cdot d} = n\cdot\frac{\lambda\cdot D}{d}.$$

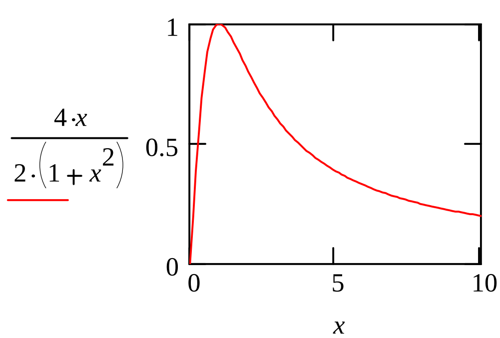
The visibility of the fringes is defined by

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$, and clearly $I_{max} = (A_1 + A_2)^2$, $I_{min} = (A_1 - A_2)^2$, whence

$$V = \frac{4\cdot\left(\frac{A_1}{A_2}\right)}{2\left[1 + \left(\frac{A_1}{A_2}\right)^2\right]}.$$

A sketch of the variation of this function with the ratio shows it to peak at equal amplitudes.

$x := 0, .1 .. 10$



With the given values $\lambda := 590\cdot 10^{-9}$ m and $d := 10^{-4}$ m, $D := 0.5$ m, the line spacing is

$$\frac{\lambda\cdot D}{d} = 2.95\cdot 10^{-3} \quad \text{m.}$$

A glass slip of thickness t is equivalent to inserting an extra path length $(\mu-1)t$ into that path. This extra path length is equates to a shift Y on the screen given by

$$(\mu - 1)\cdot t = \frac{Y\cdot d}{2\cdot D}$$

so with $\mu := 1.3$, $t := 10^{-5}$ m, the shift is

$$\frac{2\cdot D\cdot(\mu - 1)\cdot t}{d} = 0.03 \quad \text{m.}$$