12. The *principal points* of the lens system are the points at which the principal planes intersect the axis. The principal plane is the planar approximation to the surface formed by the locus of the points of intersection of the rays parallel to the axis and the corresponding rays refracted through the principal foci.

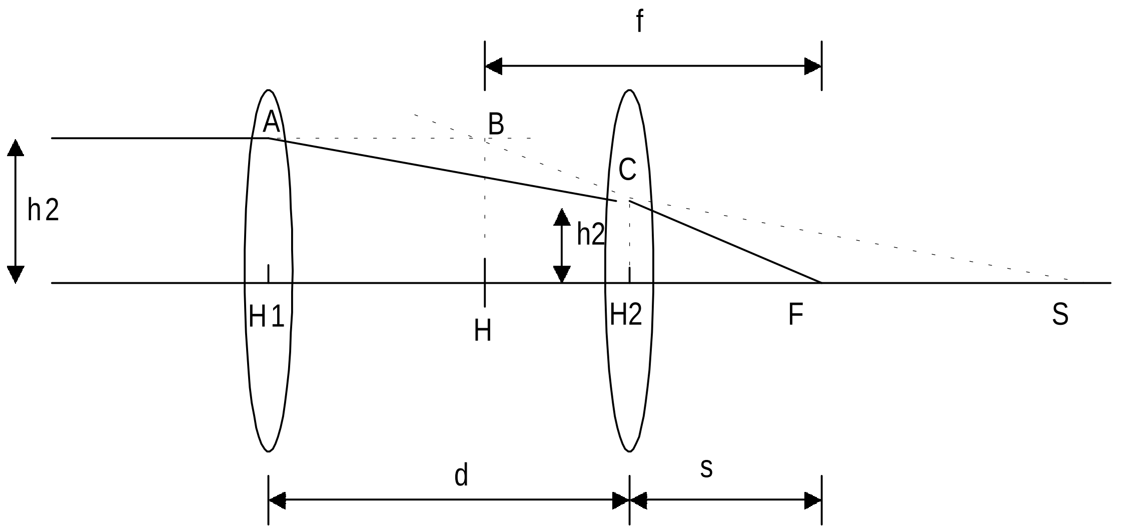
The *principal foci* of the lens system are the points at which a ray (or its projection) incident parallel to the axis of the lens system crosses the axis after passing through the system.

The *nodal points* are the points of intersection with the axis of rays which pass through the optical centre of the system, that is, rays which pass through the lens without being deviated in angle.

In the diagrams below, only one set of the foci and principal cardinal points is shown: a corresponding diagram for rays from the other side (which need not be given) will show the other set.

(2 marks each)

The diagram below sets the stage for calculating the properties of the two-lens system.



Incoming rays from the left are focused by lens 1 at point S, a distance f_1 to the left of lens 1, where the virtual source for lens 2 is therefore situated, a distance $\left(f_1 - d\right)$ to the right of lens 2. Lens 2 will then form an image at F, a distance s from lens 2, where (using the Cartesian sign convention)

$$\frac{1}{f_2} = \frac{-1}{f_1 - d} + \frac{1}{s} \qquad (1)$$

Now the initial ray entered the system at a distance h1 from the axis, and the ray leaves lens 2 at a height h2. To find the intersection point (that is. the position of the principal point H) we equate the values of h2 found by using the two pairs of similar triangles, C F H2 and B F H, C S H2 and A S H1. Note that the point F is a distance f from the principal point H:

$$h_2 = \frac{h_1}{f} \cdot s = \frac{h_1}{f_1} \cdot (f_1 - d).$$

Hence we have $s = \frac{f}{f_1} \cdot (f_1 - d)$ (2)

which may be substituted back into (1) to give

$\frac{1}{f_2} = \frac{-1}{f_1 - d} + \frac{f_1}{f \cdot (f_1 - d)}$ from which a little rearrangement yields

$f = \frac{f_1 \cdot f_2}{f_1 + f_2 - d}$. (8 marks)

If, further, we wish to locate the positions of the foci, we can recompute s from (2). The other principal focus is to the left of lens 1 by a distance $\frac{f}{f_2} \cdot (f_2 - d)$.

With $f_1 := 240$ mm and $f_2 := -80$ mm, $d := 200$ mm, the focal length of the combination is

$f := \frac{f_1 \cdot f_2}{f_1 + f_2 - d}$ or 480 mm. (2 marks)

The first focus is a distance $\frac{f}{f_1} \cdot (f_1 - d) = 80$ mm to the right of lens 2; the other focus is

$\frac{f}{f_2} \cdot (f_2 - d) = 1680$ mm to the left of lens 1. (4 marks)

(Although not asked for in the question, it is useful to note that the principal points are at $\frac{-f \cdot d}{f_1} = -400$ mm to the right of the second lens (i.e. 400 mm to the left of it) and at $\frac{f \cdot d}{f_2} = -1200$ mm to the right of the first lens (i.e. 1200 mm to the left)).

$f := 480$