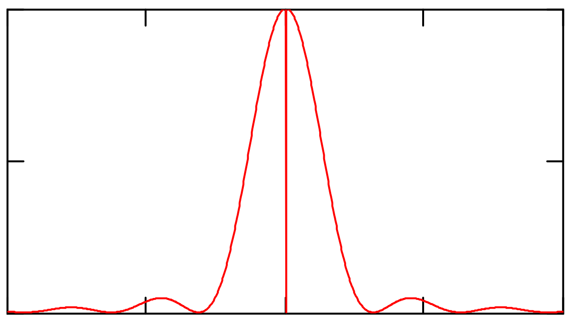
11. In Fraunhofer diffraction, both the source and the point of observation a sufficiently far from the diffracting scren that the curvature of the wavefront may be neglected. In Fresnel diffraction, the wavefront curvature is treated by including terms of second order in the distance off-axis in the phase, but not in the amplitude, of the contributing waves. (2 marks)

The sketch should show that very close to the slit geometric optics is a good approximation, with some rounding of the sharp corners. At long distances the characteristic Fraunhofer result is obtained:



Let the width of the slit w be along the y direction, with the centre of the slit at y=0, and let the long axis of the slit be along the z direction, and let the screen be perpendicular to the x axis a distance x from the slit. Then the distance r from an element of the slit of width dy at y on the y axis to an observation point at y coordinate Y on the screen is given by

$$r^2 = x^2 + (Y - y)^2$$

and the distance D from the centre of the slit to the point of observation is

$$D^2 = x^2 + Y^2$$

so that, expanding, and neglecting terms in the square of y,

$$r = D - \frac{y \cdot Y}{D}.$$

Now for small values of Y, Y/D is the sine of the angle of the point of observation away from the axis, sin(θ), so neglecting the variation of amplitude of the wave with distance the total amplitude observed at the angle θ will be

$$A(\theta) = C \cdot \int_{\frac{-w}{2}}^{\frac{w}{2}} e^{i \cdot y \cdot k \cdot \sin(\theta)}\, dy = \frac{C}{i \cdot k \cdot \sin(\theta)} \cdot 2 \cdot i \cdot \sin\left(\frac{w \cdot k \cdot \sin(\theta)}{2}\right).$$

where C involves constants and the constant phase shift exp(i k D).
The intensity, then, is

$$I(\theta) = C^2 \cdot w^2 \cdot \left[\frac{\sin\left(\frac{w \cdot k \cdot \sin(\theta)}{2}\right)}{w \cdot k \cdot \frac{\sin(\theta)}{2}}\right]^2$$

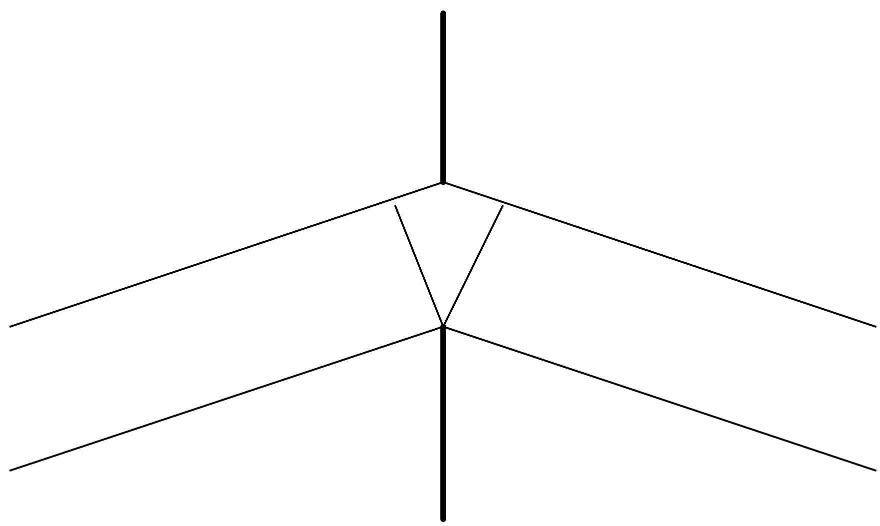
and as the term in square brackets is unity when θ is zero, we may write this as

$$I(\theta)=I(0)\cdot\left(\frac{\sin(\beta)}{\beta}\right)^2$$

with $\beta=\frac{w\cdot k\cdot\sin(\theta)}{2}=\frac{w\cdot\pi\cdot\sin(\theta)}{\lambda}$. (6 marks)

The first subsidiary maximum occurs when the argument of the sin is 3π/2, the second at 5π/2. Thus the first subsidiary maximum has relative intensity $\left(\frac{2}{3\cdot\pi}\right)^2=0.045$, the second $\left(\frac{2}{5\cdot\pi}\right)^2=0.016$. (3 marks)

If the incident light encounters the slit at an angle ϕ, the change in path length will contain an additional contribution:



So that the original integral in the derivation above is replaced by

$$A(\theta)=C\cdot\int_{\frac{-w}{2}}^{\frac{w}{2}} e^{i\cdot y\cdot k\cdot(\sin(\theta)-\sin(\phi))}\,dy=\frac{C}{i\cdot k\cdot(\sin(\theta)-\sin(\phi))}\cdot 2\cdot i\cdot\sin\left[\frac{w\cdot k\cdot(\sin(\theta)-\sin(\phi))}{2}\right]$$

and this carries over into the expression for the intensity as

$$I(\theta)=I(\phi)\cdot\left(\frac{\sin(\gamma)}{\gamma}\right)^2$$

where

$\gamma=\frac{w\cdot\pi\cdot(\sin(\theta)-\sin(\phi))}{\lambda}$. (5 marks)