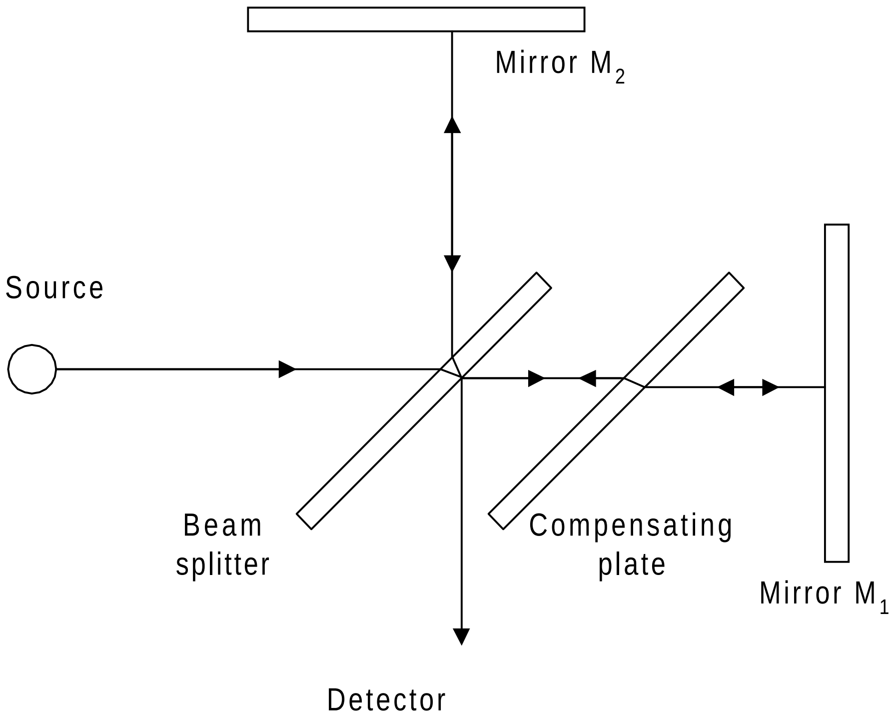
10. The Michelson interferometer




3 marks for diagram

As drawn, the mirrors $M_1$ and $M_2$ are silvered on their front faces, and the beam splitter consists of a glass plate part-silvered on its rear surface.

The glass in the beam splitter will be dispersive. This plate, however, is traversed thrice by the beam which traverses the path (source-beam splitter-$M_2$-beam splitter-detector) but only once by the beam (source-beam splitter-$M_1$-beam splitter-detector). This results in a wave-length dependent optical path difference. If the compensator plate is identical (apart from any silvering) to the beam splitter, and is aligned parallel to it, it will introduce an extra path length into the second path which is exactly equivalent to the two extra traverses of the beam splitter in the first path. (2 marks)

Circular fringes will be observed at the detector if the reflection of $M_1$ in the beam splitter is parallel to $M_2$. If one mirror is tilted, then the circular fringes will be displaced sideways until the detector sees the outer circular fringes, where the radius of curvature is large enough for them to appear straight. (2 marks)

If the reflection of $M_1$ in the beam splitter is a distance $d$ from $M_2$, and the mirrors are parallel, the geometrical path difference between a beam at an angle θ reflected from $M_1$ and one at the same angle reflected from $M_2$ will be $2 \cdot d \cdot \cos(\theta)$ . There is one extra reflection from an optically more dense medium in the path via mirror 1, corresponding to a phase change of π, so if the geometrical path length is an integral number of wavelengths $p \cdot \lambda$ then dark fringes will be observed when

$\cos(\theta) = p \cdot \frac{\lambda}{2 \cdot d}$. (3 marks for formula, 1 for stating that fringe is dark)

With a doublet, clear fringes will only be seen if $2 \cdot d \cdot \cos(\theta)$ is an integer number of wavelengths for both components of the doublet. Suppose we have a clear fringe in the centre of the field of view: then

$2 \cdot d = p_1 \cdot \lambda_1 = p_2 \cdot \lambda_2$, or

$$p_1 - p_2 = 2 \cdot d \cdot \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right).$$

If d is changed to d+D to reach the next peak in visibility, then the difference in the orders must have changed by one, that is

$$p_1 - p_2 + 1 = 2 \cdot (d + \Delta) \cdot \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

and subtracting and rearranging gives

$$\lambda_2 - \lambda_1 = \frac{\lambda_1 \cdot \lambda_2}{2 \cdot \Delta}$$

in which we may replace the product of wavelengths on the right-hand side by the square of the average wavelength. (7 marks)

If we have an average wavelength $\lambda := 578 \cdot 10^{-9}$ m, and the mirror shift is $\Delta := 83 \cdot 10^{-6}$ m, the splitting is

$$\frac{\lambda^2}{2 \cdot \Delta} = 2 \cdot 10^{-9}$$ m. (2 marks)