Answer SIX questions from section A and THREE questions from section B.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

SECTION A

[Part marks]

1. Write down the partial differential equation describing wave motion in one dimension. Show that this equation is satisfied by a disturbance of the form $y_1(x,t) = A\sin(kx - \omega t)$. What is the speed of the wave in terms of k and ω? **[3]**

   This wave is superposed on another wave $y_2(x,t) = A\sin(kx + \omega t)$. Explain how the two waves differ, and describe the resulting disturbance. **[4]**

   You may find it useful to note that $\sin(a) + \sin(b) = 2\sin(\frac{1}{2}(a+b))\cos(\frac{1}{2}(a-b))$.

2. State the changes of phase which occur when light is reflected a) in air from an air-glass interface, b) in glass from a glass-air interface. **[2]**

   A plano-convex lens is placed with its curved surface with radius of curvature R resting on a plane glass surface. Newton's rings are observed by illuminating from above with light of wavelength λ. Draw a diagram showing how the fringes are formed, and show that the radii of the dark rings are given by $r = \sqrt{nR\lambda}$, where n is an integer. **[5]**

   [If a diameter of a circle of radius R intersects a chord of length $2r$ at right angles, dividing the diameter into two lengths $2R - h$ and h, then $2Rh \approx r^2$ for small h.]

3. Write down an expression for the specific acoustic impedance Z in terms of the displacement velocity and excess pressure in a plane sound wave in a gas. **[2]**

   The transmitted displacement amplitude when a longitudinal sound wave of unit amplitude is incident normally from a medium with acoustic impedance Z_1 on an interface with a medium with acoustic impedance Z_2 is

   $$t = \frac{2Z_1}{Z_1 + Z_2}.$$

   Write down the corresponding expression for the reflected amplitude. The energy flux density in the sound wave may be written in terms of its displacement amplitude ξ_0 and angular frequency ω as $\frac{1}{2}\xi_0^2\omega^2 Z$: show that energy is conserved at the interface. **[5]**

4. Show that the frequency f' measured by a stationary observer of a source of sound which is approaching at a speed v_s whilst emitting a constant tone of frequency f is

$$f' = \frac{f}{1 - v_s/v}$$

where v is the velocity of sound. **[4]**

When a train is approaching a stationary observer its hooter has an apparent frequency of 320 Hz, but while it is receding the apparent frequency is 280 Hz. If the speed of sound in air is 330 m s^{-1}, at what speed is the train travelling? **[3]**

5. Show by means of a ray diagram how a thin converging lens may be used as a simple magnifying glass. **[4]**

What focal length must the thin lens have if it is to give a magnification factor of 5 with the image at infinity? You may assume that the near point distance for normal vision is 250 mm. **[3]**

6. An ideal astronomical interferometer comprises a number of identical aerials laid out with a regular spacing a along a straight base line. Show that a distant star will be observed clearly if it lies at an angle θ_n to the perpendicular to the line of aerials given by

$$a \sin\theta_n = n\lambda,$$

where n is an integer and λ is the wavelength of the signal at the aerials. **[4]**

In the first such interferometer the aerials were spaced 7 m apart and were used to observe stars using the 210 mm hydrogen emission line. At what angle to the normal to the base line would the first minimum in the received signal occur? **[3]**

7. Draw a diagram of a Fabry-Perot interferometer, including a source, screen, and any necessary optical elements. **[5]**

Sketch the pattern that would be observed on the screen with a monochromatic source. **[2]**

8. What is meant by *phase velocity* and by *group velocity*? **[2]**

Show that the group velocity v_g may be written in terms of the phase velocity v_p and the wavevector k as

$$v_g = v_p + k\frac{dv_p}{dk}.$$ **[3]**

The phase velocity of ocean waves with long wavelength λ is given by

$$v_p^2 = \frac{g\lambda}{2\pi},$$

where g is the acceleration due to gravity and ρ is the density of the water. Show that the group velocity of these waves is less than their phase velocity. **[2]**

SECTION B

9. Show that the speed of longitudinal waves on a rod of uniform cross-section with density ρ and Young's modulus Y is given by $v = \sqrt{Y/\rho}$. **[5]**

   Two rods of equal cross-section, characterised by Y_1, ρ_1 and Y_2 and ρ_2, are joined end-to-end. Write down the conditions which must be satisfied at the join. Hence show that the amplitude transmission coefficient from rod 1 to rod 2 is

$$t = \frac{2\sqrt{Y_1\rho_1}}{\sqrt{Y_1\rho_1} + \sqrt{Y_2\rho_2}},$$

   and derive the corresponding reflection coefficient. **[6]**

   Calculate the amplitudes of the reflected and transmitted waves when a longitudinal wave with an amplitude of 1 nm in a perspex rod (density 1200 kg m^{-3}, Young's modulus 3 GN m^{-2}) encounters a join with a rod of the same cross-section made of steel (density 7900 kg m^{-3}, Young's modulus 200 GN m^{-2}), and comment on the sign of the reflected amplitude. **[5]** State how the transmission of sound energy between two rods with different physical properties may be maximised. **[4]**

10. Draw a diagram of a Michelson spectral interferometer. Include in your diagram a compensating plate, and explain why it is important for observing fringes with white light. Describe how the interferometer should be set up to observe a) circular fringes and b) straight fringes. **[7]**

    Show that when the interferometer is set up to show circular fringes, the angles at which such fringes are observed are given by

$$\cos(\theta) = n\frac{\lambda}{2d}.$$

    Explain the significance of the symbols in this formula and state whether these fringes are dark or light. **[4]**

    The interferometer is used to study the doublet emission line of mercury which is centred on 578 nm. Initially it is set up for maximum fringe visibility: when the mirror is moved the fringes become less distinct, but are clearly visible again when the mirror has been moved by 83 μm. Explain this behaviour, and calculate the splitting of the doublet. **[9]**

11. Explain the difference between Fresnel and Fraunhofer diffraction. A slit of width w is illuminated by collimated light of wavelength λ which is incident normal to the plane containing the slit, and the transmitted light is observed on a screen parallel to that plane. Sketch the pattern that will be observed on the screen a) when it is very close to the slit and b) when it is far from the slit. **[6]**

Show that in the Fraunhofer regime the pattern of intensity observed in the above experiment is

$$I(\theta) = I(0)\left[\frac{\sin(\beta)}{\beta}\right]^2,$$

stating clearly the relationship of β to λ and w. Hence calculate the ratios of the intensities of the first and second subsidiary maxima to that of the central peak for a single slit. **[8]**

When N similar parallel slits are ruled with their centre lines a distance d apart, to form a diffraction grating, the expression for the intensity as a function of angle becomes

$$I(\theta) = I(0)\left[\frac{\sin(\beta)}{\beta}\right]^2\left[\frac{\sin(N\alpha)}{N\sin\alpha}\right]^2,$$

where $\alpha = (\pi d\sin\theta)/\lambda$. Such a grating is ruled with 500 lines/mm and with the width of each slit equal to half the separation d: calculate the angular positions of the principal maxima that may be observed when the wavelength of the incident light is 600 nm. **[6]**

12. Explain, with the aid of diagrams, what are meant by the *principal points*, *principal foci* and *nodal points* of a thick lens system. **[6]**

Draw a ray diagram of a telephoto lens. **[2]**

An optical system is formed from two thin lenses of focal lengths f_1 and f_2 a distance d apart. Show that the focal length of the combination is

$$f = \frac{f_1 f_2}{f_1 + f_2 - d}.$$

[6]

If one lens is converging with a focal length of 240 mm, the other is diverging with a focal length of 80 mm, and they are 200 mm apart, calculate the focal length and the positions of the two foci relative to the thin lenses. **[6]**

13. Explain what is meant by *coherence* of a wave pattern, and describe two ways of forming two coherent sources from a single incandescent source. **[6]**

In an experiment to show Young's fringes, two narrow parallel lines with their centres a distance d apart are scratched through the coating of a blackened photographic plate. If the slits are illuminated through a single source slit with light of wavelength λ and the resulting pattern is observed on a screen a distance x from the photographic plate, derive an expression for the positions of the bright lines on the screen. **[4]**

The slits are of similar width, but are scratched to different depths so that the amplitudes A_1 and A_2 transmitted are different: derive an expression for the visibility of the fringes. **[4]**

The experiment described above is conducted with sodium light (wavelength 590 nm) and the scratches are 0.1 mm apart. If the screen is 500 mm from the plate, what is the separation of the bright lines? What is the visibility of the fringes (i) if the amplitudes from the two slits are equal, (ii) if one amplitude is twice as large as the other? **[6]**

END OF PAPER