8pt **Answer SIX questions from section A and THREE questions from section B.**

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

speed of light $c = 3 \times 10^8$ m s^{-1}

SECTION A **[Part marks]**

1. The excess pressure in a plane sound wave in air has the form

$$p(x,t) = 2\cos(0.1\pi x - 33\pi t)\sin(1.7\pi x - 561\pi t),$$

where x is measured in metres and t in seconds. Show that this sound may be described as a superposition of two sinusoidal waves, and calculate the frequencies of the two **[4]** waves and the speed of sound in air. **[3]**

2. A uniform film of oil of refractive index $n_{\rm oil} = 1.44$ and $h = 0.27$ μm thick covers a puddle of water (refractive index $n_{\rm water} = 1.33$). Draw a diagram illustrating the occurrence of interference in reflected light observed from almost vertically above the **[3]** puddle. What is the change of phase of the light on reflection in air from the upper surface of the oil? Explain why constructive interference will be observed for light with **[1]** wavelength λ in air which satisfies

$$\lambda = \frac{4n_{\rm oil}h}{2p+1}$$

where p is a positive integer. Estimate the wavelength of light in the visible range (wave- **[2]** lengths 0.4 to 0.7μm) for which the observed intensity will be a maximum. **[1]**

3. Write down an expression for the velocity of sound in a gas in terms of its adiabatic bulk **[1]** modulus $B_{\rm a}$ and density ρ. Explain why it is usually appropriate to use the adiabatic bulk modulus in this context. The specific acoustic impedance of a gas is $Z = \sqrt{B_{\rm a}\rho}$, **[2]** and the amplitude transmission coefficient for sound from a gas with specific acoustic impedance Z_1 to one with specific acoustic impedance Z_2 is $t = 2Z_1/\left(Z_1 + Z_2\right)$. Discuss the limiting case of very large Z_2. Calculate the fraction of the amplitude of sound **[2]** which will be transmitted from air to helium, both at normal temperature and pressure, given that the respective densities are 1.18 and 0.18 kg m^{-3}. The adiabatic bulk modulus **[2]** of a perfect gas at pressure p is γp. For air γ, the ratio of the specific heats at constant pressure and at constant volume, is equal to 7/5: for helium it is 5/3.

4. The formula relating the focal length of a biconvex lens in air to the radii of curvature of the surfaces of the lens and the refractive index of the lens is

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

Explain the significance of the terms in this formula, and define the sign convention that has been used. **[2]**

A lens for a microwave system is made by embedding metal spheres in expanded polystyrene to give a material with a refractive index of 12. If a symmetrical biconvex lens made from this material is used to produce a parallel beam from a source placed on the axis of the lens and two metres from it, what is the radius of curvature of the two surfaces? **[3]** Sketch a ray diagram of the system, showing clearly the refraction at each surface of the lens. **[2]**

5. Two narrow parallel slits a distance d apart are illuminated by a slit parallel to and equidistant from them with light of wavelength λ in air, and the transmitted light is observed on a screen perpendicular to the line joining the first slit to a point midway between the two slits and a distance D away ($D >> d$) from that midpoint. What is the separation of successive bright bands on the screen? **[5]**

How will the result be affected if the space between the slits and the screen is filled with a medium of refractive index n? **[2]**

6. Explain, with the aid of a diagram, Rayleigh's criterion for the resolution of the images of two identical slit sources under Fraunhofer conditions. **[3]** State the corresponding condition for the resolution of two point sources imaged with a circular lens. **[1]**

A hobbyist's reflecting telescope has a primary mirror 200 mm in diameter. What is the smallest angular separation of two stars that can be resolved using light with a wavelength of 500 nm? **[3]**

7. Explain what is meant by the terms *phase velocity* and *group velocity*. **[2]** Show that the phase velocity $v_{\rm p}$ and group velocity $v_{\rm g}$ are related by

$$v_{\rm g} = v_{\rm p} + k\frac{{\rm d}v_{\rm p}}{{\rm d}k}$$

where k is the wave- vector. **[3]**

The phase velocity $v_{\rm p}$ of light in glass may be written as $v_{\rm p} = A + (B/k^2)$, where A and B are constants. Show that the group velocity is $v_{\rm g} = A - (B/k^2)$. **[2]**

8. Draw a diagram of a Michelson interferometer, and label its main features. Under what conditions will circular fringes be observed with the apparatus you have drawn? **[4]**

Explain why a compensator plate is necessary if a Michelson interferometer is to be used with non-monochromatic light. **[3]**

9. Describe the main features of a Fabry-Perot interferometer, illustrating your answer with a diagram of the system and a sketch of the observed interference pattern. Derive an expression for the half-width of the interference maxima. Why is it possible to obtain a stable interference pattern with illumination from an extended source? **[6]** **[8]** **[3]**
Explain qualitatively how dielectric coatings may be used to improve the performance of a Fabry-Perot etalon. **[3]**

10. Explain with the aid of a diagram what is meant by the terms *focal points*, *principal points* and *nodal points* for a thick lens system. **[6]**
Two thin lenses with focal lengths f_1 and f_2 are mounted coaxially a distance d apart in air. Show that the focal length f of the resulting system is given by **[6]**

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}.$$

The eyepiece of an optical instrument is formed from two converging lenses 40 mm apart, with focal lengths 60 mm and 30 mm, the latter being closer to the eye. Find the focal length of the eyepiece and the positions of the principal points and the focal points. **[8]**

11. State Huygens's principle in the propagation of light, explain under what conditions it leads to Fraunhofer's expression for diffraction, and write down that expression. **[5]**
A diffraction grating comprises N identical parallel slits in an opaque screen. The slits are of negligible width, and their centres are h apart. Show that in the Fraunhofer regime the intensity distribution function as a function of angle θ for light of wavelength λ is

$$I(\theta) = I(0)\left(\frac{\sin N\gamma}{N\sin\gamma}\right)^2,$$

where $\gamma = (\pi h/\lambda)\sin\theta$. **[9]**
If the slits are taken to have widths b, this expression is modified to

$$I(\theta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\gamma}{N\sin\gamma}\right)^2,$$

where $\beta = (\pi b/\lambda)\sin\theta$.
Sketch graphs showing how the diffraction pattern is affected by a) the number of slits, each of which is very narrow b) the width of the slits, when the number of slits is large. **[3]** **[3]**

12. Show that the speed, v, of transverse waves on a string of linear density ρ held under a tension T is given by $v^2 = T/\rho$. **[6]**

Such a string has its ends, a distance L apart, held rigidly. Show that the string will only vibrate freely at certain frequencies, and derive an expression for these normal mode frequencies in terms of L, T and ρ. **[4]**

Derive an expression for the total kinetic energy of the string in the lowest normal mode, and use the facts that the maximum potential energy in the mode is equal to the maximum kinetic energy in the mode, and that the total energy in the mode is independent of time, to show that if the amplitude of the vibration is A the total energy in the lowest mode is $\pi^2 A^2 T/(4L)$. **[6]**

A string with a linear density of $2.5\ \mathrm{g\ m^{-1}}$ is under a tension of $5\ \mathrm{N}$ with its ends fixed. If it is oscillating in its fundamental mode at a frequency of 400 Hz with an amplitude of 10 mm, how much vibrational energy is stored in the string? **[4]**

13. Show that in the nonrelativistic regime the Doppler formula relating the frequency f_{obs} detected by an observer moving in free space at velocity v_{o} away from a source which is emitting radiation at a constant frequency f while it is moving with velocity v_{s} away from the observer through a medium in which the wave speed is c' is given by

$$f_{\mathrm{obs}} = f\left(\frac{c - v_{\mathrm{o}}}{c' + v_{\mathrm{s}}}\right)\frac{c'}{c}.$$ **[7]**

A sounding rocket with a stable 10 MHz radio transmitter is sent into the ionosphere (where the refractive index may be less than unity). The rocket is moving vertically upwards at $600\ \mathrm{m\ s^{-1}}$, and the frequency of the signal detected at the ground station directly below it is measured by beating it with another stable 10 MHz signal. The beat frequency is 10 Hz. What is the refractive index at a frequency of 10 MHz of the medium in which the rocket is moving? **[6]**

The relativistic result for the observed frequency f' when a source emitting a frequency f and an observer move apart at a relative velocity v in free space is

$$f' = f\sqrt{\frac{c - v}{c + v}}.$$

Show that the nonrelativistic result agrees with this to first order in v/c, and estimate the fractional error in the refractive index computed above as a result of using the nonrelativistic formula. **[4]** **[3]**

END OF PAPER