Answer SIX questions from section A and THREE questions from section B.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

SECTION A **[Part marks]**

1. State the *principle of superposition*, and use it to calculate the disturbance $y(x,t)$ resulting from the presence in a medium of two waves $y_1(x,t) = A\ \mathrm{Re}\left[e^{i(\omega t - kx)}\right]$ and $y_2(x,t) = A\ \mathrm{Re}\left[e^{i(\omega t + kx)}\right]$, where $\mathrm{Re}[z]$ denotes the real part of z. **[4]**

   State the physical difference between y_1 and y_2, and explain the significance of the result you have calculated. **[3]**

2. Show that an ideal diffraction grating with narrow slits spaced a distance d apart illuminated with light of wavelength λ will produce an intensity pattern with peaks at angles θ given by **[4]**
   $$d \sin(\theta) = n\lambda,$$
   where n is an integer.

   If such a diffraction grating with 500 slits per mm is illuminated with 600 nm light, what is the maximum order of diffraction, n, that will be visible? **[3]**

3. State the change of phase which occurs when a light wave is reflected from (i) an optically more dense material, (ii) an optically less dense material. **[2]**

   Draw a diagram showing the formation of interference rings in Newton's experiment, in which a plano-convex lens is placed with its surface of radius of curvature R on a plane surface. Prove that the radii r_n of the bright rings formed with light of wavelength λ are given by **[5]**
   $$r_n = \sqrt{(2n+1)\frac{\lambda}{2}R}.$$

4. The intensity in a sound wave of amplitude A and angular frequency ω propagating in a fluid of specific acoustic impedance Z is $\frac{1}{2}ZA^2\omega^2$. If the coefficients of reflection and transmission for a wave incident normally on an interface between two materials with impedances Z_1 and Z_2 are
   $$r = \frac{Z_1 - Z_2}{Z_1 + Z_2} \quad \text{and} \quad t = \frac{2Z_1}{Z_1 + Z_2},$$
   show that energy is conserved at the interface. **[5]**

   The specific acoustic impedance of air is $400\ \mathrm{kg\ m^{-2}\ s^{-1}}$, that of water is $1.45 \times 10^6\ \mathrm{kg\ m^{-2}\ s^{-1}}$. What is the amplitude transmitted to the water if a sound wave of amplitude 10^{-6} m is incident from air? **[2]**

5. Show that the frequency f' measured by a stationary observer from a source which emits sound at a constant frequency f but which is approaching the observer at a speed $v_{\rm s}$ in a medium in which the speed of sound is c is **[4]**

$$f' = \frac{f}{1 - (v_{\rm s}/c)}.$$

Hence show that the difference in observed frequency between the source approaching at a speed $v_{\rm s}$ and receding at the same speed is **[3]**

$$\Delta f = \frac{2f(v_{\rm s}/c)}{1 - (v_{\rm s}/c)^2}.$$

6. Draw a ray diagram showing the formation of the image of a slide on a screen by a simple converging lens. **[3]**

A thin converging lens of focal length 50 mm is to be used to form an image of a slide on a screen 5 m from the lens. How far from the lens should the slide be placed? If the slide is 24 mm high, how high will the image be? **[4]**

7. Draw a clearly labelled diagram of a Michelson interferometer. **[3]**

A Michelson interferometer is set up to observe straight fringes with light of wavelength 600 nm, in order to observe small displacements of the movable mirror. 100 dark fringes are observed to pass the cross-hairs in the eyepiece. Through what distance has the mirror moved? **[4]**

8. Write down expressions for the phase velocity $v_{\rm p}$ and group velocity $v_{\rm g}$ of a wave in terms of the angular frequency ω and the wavevector k. At what speed does energy propagate in the wave? **[3]**

The dispersion relation for transverse waves on an elastic beam is

$$\omega = \frac{a}{\lambda^2},$$

where λ is the wavelength and a is a constant depending on the material and dimensions of the beam. Show that for these waves the group velocity is twice the phase velocity. **[4]**

SECTION B

9. Derive from first principles the equation of motion for transverse waves on a uniform string with mass per unit length ρ under tension T, and hence show that the speed of waves on the string is given by $v = \sqrt{T/\rho}$. **[7]**

   Derive an expression for the kinetic energy contained in one wavelength of such a transverse wave of amplitude A and angular frequency ω. Hence show that the average kinetic energy per unit length may be written as

$$\langle E_{\rm k} \rangle = \frac{1}{4}\rho\omega^2 A^2.$$

   Use this result and the fact that the average potential energy per unit length $\langle E_{\rm p} \rangle$ is equal to $\langle E_{\rm k} \rangle$ to derive an expression for the rate of energy transport along the string. **[6]**

   If waves of amplitude 10 cm are to be transmitted along a string of linear density 10 g m^{-1} which is under a tension of 100 N, what is the highest frequency $f_{\rm max}$ achievable if the source can deliver a maximum of 1 kW? If the same string under the same tension is held between two fixed points, how far apart must those points be if the lowest standing wave frequency is equal to $f_{\rm max}$? **[7]**

10. Draw a diagram showing the operation of a Fabry-Perot interferometer, labelling it clearly and emphasising features which are important in determining the resolution of the instrument. **[4]**

    Show that the amplitude transmitted through the etalon may be written as

$$E_t = E_0 \left[\frac{tt'}{1 - rr'e^{i\delta}} \right],$$

    explaining the significance of the symbols. **[7]**

    Hence derive an expression for the transmitted intensity I_t in terms of the coefficient

$$F = \frac{4rr'}{(1 - rr')^2},$$

    and sketch the intensity pattern for different values of F. Discuss your sketch in terms of the design of the etalon. **[9]**

11. Draw a diagram illustrating Young's experiment for generating interference fringes from a spatially incoherent source. Show that if the apparatus is set up with two identical slits with centres d apart, the pattern formed on a screen a distance x from the slits with light of wavelength λ will have an intensity

$$I(y) = I_0 \left[\cos \left(\frac{\pi d y}{\lambda x} \right) \right]^2 ,$$

making clear any approximations made. **[8]**

Explain how the maximum and minimum intensities in the interference pattern are related to the amplitudes of the waves from the two slits. Hence derive an expression for the visibilty of the fringe pattern if the slits pass amplitudes A and B. **[6]**

A slip of glass with refractive index 1.5 is placed over one of the slits, but the positions of the peaks near the centre of the interference pattern formed with 500 nm light appear to be unchanged. What are the possible thicknesses of the glass? **[6]**

12. Describe, with the aid of diagrams, two types of *aberration* which may be found in lens systems, and for one of the examples you choose suggest how the system may be modified to correct for it. **[6]**

A compound microscope is formed from two converging lenses, an objective with focal length 4 mm and an eyepiece with focal length 20 mm. The microscope is adjusted by altering the separation of the lenses. Draw a ray diagram (not necessarily to scale) showing the formation of an image 250 mm from the eye of an object placed 4.1 mm from the objective. **[5]**

What must the separation of the lenses be in this setting? What is the magnification of the microscope when adjusted in this way? Why is it more usual to adjust the microscope so that the image is at infinity? **[9]**

13. Describe the difference between the conditions under which Fraunhofer and Fresnel diffraction may be observed. Show that the intensity distribution in the Fraunhofer pattern of a slit of width w illuminated with light of wavelength λ is

$$I(\theta) = I(0)\left[\frac{\sin(\beta)}{\beta}\right]^2,$$

where **[9]**

$$\beta = \frac{\pi w}{\lambda}\sin(\theta).$$

Describe Rayleigh's criterion for the resolution of images formed by a slit, and deduce from the above formula for the diffraction pattern that the minimum angular separation between two images which can just be resolved, at wavelength λ, by a slit of width w, is λ/w. State how this expression is modified for a circular aperture of diameter D. **[7]**

Use this result to calculate the smallest separation between two objects that can be resolved by a human eye with a pupil diameter of 2.5 mm at a distance of 250 mm, assuming a wavelength of 500 nm. **[4]**

 END OF PAPER