Answer SIX questions from section A and THREE questions from section B.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

SECTION A

[Part marks]

1. Calculate the disturbance $y(x,t)$ resulting from the superposition of two waves $y_1(x,t) = A\sin(k_1x - \omega_1t)$ and $y_2(x,t) = A\sin(k_2x - \omega_2t)$. Sketch the resulting disturbance *and* the intensity as functions of time at $x = 0$. **[5]**

   Explain how your diagrams explain the phenomenon of beats. **[2]**

   You may find it useful to note that $\sin(a) + \sin(b) = 2\sin(\frac{1}{2}(a+b))\cos(\frac{1}{2}(a-b))$.

2. Define the *visibility* of an interference pattern. The two apertures in a Young's slits experiment pass different amplitudes A and B. Show that the visibility of the resulting fringe pattern may be written as **[4]**

$$V = \frac{2A/B}{1 + A^2/B^2}.$$

   What will the visibility be if the *intensity* passed by one of the slits is 4 times that passed by the other? **[3]**

3. State the changes of phase which occur when light is reflected (a) in air from an air-water interface, (b) in water from a water-air interface. **[2]**

   White light is incident normally on a soap film with a refractive index of 4/3. If the film is 0.75 μm thick, for what wavelengths in air will maxima be seen in the intensity of the reflected light? Take the visible range as being from 400 to 700 nm. **[5]**

4. Write down an expression for the speed of sound in air in terms of its equilibrium density ρ_0 and adiabatic bulk modulus B_{a}. Explain why it is appropriate to use the adiabatic bulk modulus rather than the isothermal bulk modulus. **[3]**

   Sketch the variation of displacement along a tube, closed at both ends, in which air is vibrating at its fundamental frequency. Calculate the fundamental frequency if the tube is 1 m long and the speed of sound in air is $330\ \mathrm{m\ s^{-1}}$. **[4]**

5. Show that the frequency f' measured by an observer who is approaching, at a speed v_{o}, a stationary source which emits sound at a constant frequency f is

$$f' = f(1 + v_{\mathrm{o}}/v),$$

where v is the velocity of sound. State the full expression describing the situation when the source moves towards the observer at a speed v_{s} *and* the observer approaches the source at a speed v_{o}. **[5]**

A police car emits a sound signal of 500 Hz. A stationary observer measures the frequency as 525 Hz while the car is approaching. What is the observed frequency when the car recedes at the same speed? **[2]**

6. Draw a ray diagram showing the formation of a magnified image in a concave spherical mirror. **[3]**

A shaving or make-up mirror of this type has a radius of curvature of 30 cm. What is the magnification of the image when the face is 10 cm from the centre of the mirror? **[4]**

7. Explain, with the aid of a diagram, Rayleigh's criterion for the resolution of two diffraction patterns. **[3]**

Two slit sources, viewed through an aperture of width w, are resolvable if their angular separation is greater than λ/w. How is this result modified in the case of point sources and a circular aperture? If an Earth observation satellite orbits at an altitude of 500 km and its camera has an aperture of diameter 1 cm, what is the smallest separation of two objects on the Earth's surface that could be resolved at a wavelength of 500 nm? **[4]**

8. Explain the difference between *phase velocity* and *group velocity*. **[2]**

Show that the group velocity v_{g} may be written in terms of the phase velocity v_{p} and the wavevector k as

$$v_{\mathrm{g}} = v_{\mathrm{p}} + k\frac{\mathrm{d}v_{\mathrm{p}}}{\mathrm{d}k}.$$ **[2]**

The phase velocity v_p of light in glass varies with wavelength λ as $v_p = A + B\lambda^2$, where A and B are constants. Show that the sum of the group velocity and the phase velocity is constant in the glass. **[3]**

SECTION B

9. Derive from first principles the equation of motion for longitudinal waves in a uniform rod of density ρ and Young's modulus Y, and hence show that the speed of sound in the rod is $v = \sqrt{Y/\rho}$. **[7]**

   Two semi-infinite rods with equal cross-sections are joined together end-to-end. The rod on the left has density ρ and Young's modulus Y, that on the right is characterised by ρ' and Y'. State the boundary conditions which hold at the interface, and derive the amplitudes of waves transmitted and reflected at the interface when a wave is incident in the rod on the left. **[7]**

   Derive an expression for the kinetic energy density in a rod carrying a longitudinal wave of amplitude A and angular frequency ω. Hence show that the kinetic energy density averaged over a period may be written as

$$\langle T \rangle = \frac{1}{4}\rho\omega^2 A^2.$$

   What is the average kinetic energy density of a 1 kHz wave with amplitude 10^{-8} m in a metal bar with density 8000 kg m^{-3}? **[6]**

10. Draw a diagram of a Michelson interferometer: include a compensating plate and explain its importance. **[7]**

   If the amplitudes of the signals travelling along the two paths are equal, show that the resulting intensity distribution of the interference fringes may be written as

$$I(\theta) = I(0)\sin^2\left(\frac{2\pi}{\lambda} d\cos(\theta)\right).$$

   Explain the significance of all the quantities in this expression. **[5]**

   Explain how the interferometer may be set up to observe either circular or straight fringes. **[3]**

   The interferometer is set up to show straight fringes using light of wavelength 500 nm with an evacuated cylinder of length 50 mm in one arm. Gas is slowly let into the cylinder, and the fringe pattern shifts by 50 fringes. Calculate the refractive index of the gas. **[5]**

11. Explain what is meant by *coherence* of a wave pattern, distinguishing between longitudinal and transverse coherence. **[3]**

Show that the total electric field in the Fraunhofer diffraction pattern produced from monochromatic light with wavelength λ by a grating with N narrow slits at uniform spacing d is given by

$$E = E_1 \frac{1 - e^{2iN\gamma}}{1 - e^{2i\gamma}},$$

where E_1 is the field due to one slit and **[5]**

$$\gamma = \frac{\pi d \sin(\theta)}{\lambda}.$$

Hence show that the intensity pattern may be written as

$$I(\theta) = I(0) \left[\frac{\sin(N\gamma)}{N \sin(\gamma)} \right]^2 .$$

[3]

Use this formula to show that the intensity pattern in Young's experiment has a $\cos^2(\gamma)$ form, and sketch the intensity patterns for this case and for a grating with 10 slits. Comment on the differences. **[5]**

A grating ruled with 800 lines/mm is illuminated with light of wavelength 500 nm. How many orders of diffraction will be visible? Calculate the angular positions of the principal maxima in the diffraction pattern. **[4]**

12. Explain, with the aid of diagrams, what are meant by the *principal points, principal foci* and *nodal points* of a thick lens system. **[6]**

Sketch a ray diagram illustrating the formation of an image by a telephoto lens. **[3]**

A telephoto lens is made with a diverging lens of focal length 100 mm placed between a converging lens of focal length 50 mm and the film. The two lenses are 20 mm apart. If an object is placed 500 mm in front of the converging lens how far must the film be behind the diverging lens? **[5]**

What is the ratio of the final image size to the object size in this arrangement? **[6]**

13. Describe the difference between the conditions under which Fraunhofer and Fresnel diffraction may be observed. Show that the minimum distance D at which a screen must be placed from a slit of width w in order to observe Fraunhofer diffraction with light of wavelength λ is given by $D = w^2/\lambda$. **[6]**

Show that the intensity distribution in the Fraunhofer pattern of a slit of width w illuminated with light of wavelength λ is

$$I(\theta) = I(0)\left[\frac{\sin(\beta)}{\beta}\right]^2,$$

where **[9]**

$$\beta = \frac{\pi w}{\lambda}\sin(\theta).$$

Describe in outline the factors which would lead to the selection of the optimal size of hole for a pinhole camera. **[5]**

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