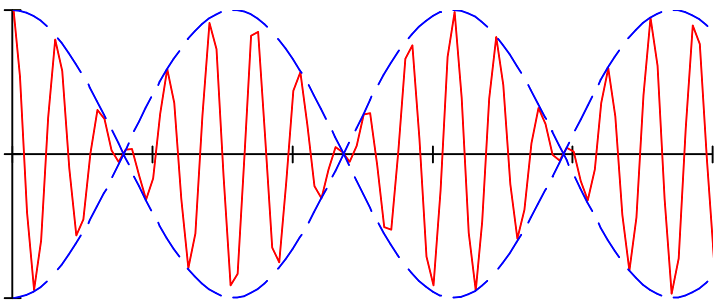
Physics 1B24 Waves, Optics and Acoustics
Model Answers

Part A

1. The wave equation is $c^2\, \partial^2 y/\partial x^2 = \partial^2 y/\partial t^2$ for wave speed c. 2 marks

Using the expression given, we have
$y_1(x,t)+y_2(x,t) = 2\,A \cos((k+k')x/2 - (\omega+\omega')t/2)\ \cos((k-k')x/2 - (\omega-\omega')t/2)$
which describes a wave characterised by a carrier and an envelope, typified by the diagram below (note that the figure is only intended to be schematic).

Derivation 2
Sketch 3



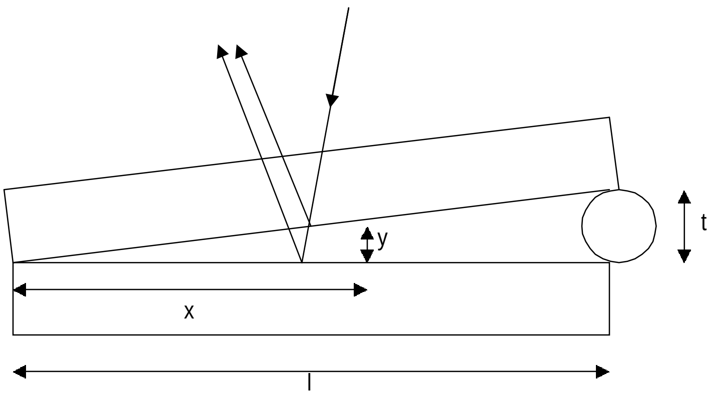
1/2 mark credit if beats mentioned but not clear in sketch.

Do not insist on y=2A at x,t=0.

2. The phase change on reflection from the air-glass interface is π,
from the glass-air interface is 0.

2 marks
allow 1/2 mark if swapped

The interference process is sketched below.



Setting up 2
derivation 2
number 1

The interference is between rays reflected from the lower surface of the upper plate and the upper surface of the lower plate. At a distance x from the contact line the plate separation is $y = x\,(t/l)$. The separation s of the dark fringes corresponds to a change in path length of one wavelength λ, or a change in y of $\lambda/2$.

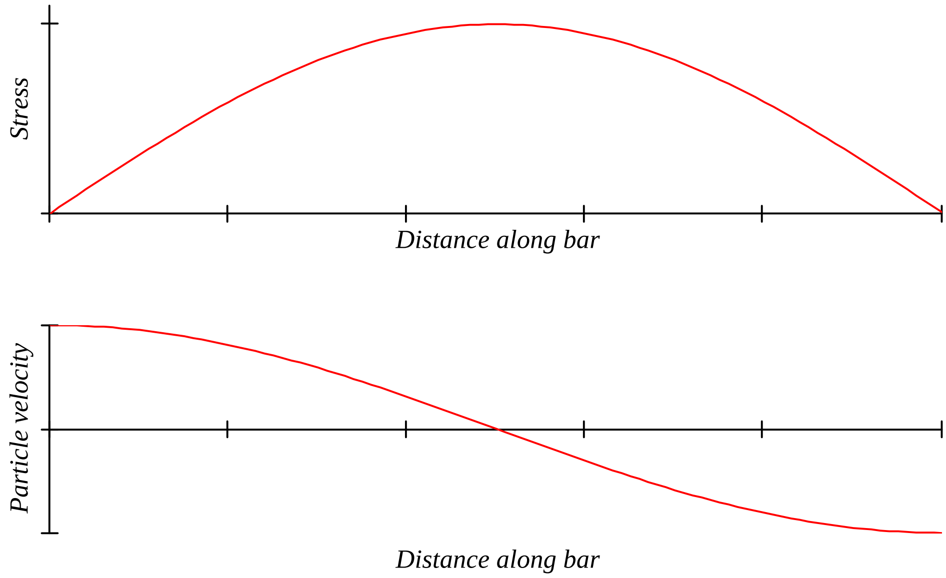
Hence $\lambda/2 = s\,(t/l)$, or $t = \lambda\, l/(2\,s)$.

In this case $t = \dfrac{600\cdot 10^{-9}\cdot 100\cdot 10^{-3}}{2\cdot 2\cdot 10^{-3}} = 1.5\cdot 10^{-5}$ m is the thickness of the hair.

2 marks
Allow 1 for inverted form, square root omitted etc.

3. Longitudinal waves in a solid rod travel at a speed $c = \sqrt{\frac{Y}{\rho}}$.

If the bar is freely suspended, the stresses at the ends are zero. The velocity, however, has maxima at the ends. In the lowest mode of vibration, one half-wavelength fits into the length, so we have



1 for each graph
Make allowance for getting correct phase relation between stress and velocity.

1 mark each for λ, velocity, result.

We know, then, that the wavelength λ is 2m, and the wave speed is $\sqrt{\frac{150\cdot 10^9}{6000}} = 5\cdot 10^3$ m/s,

so the frequency is velocity/λ = $\frac{5\cdot 10^3}{2} = 2.5\cdot 10^3$ Hz.

4. The stationary source emits waves with a constant wavelength $\lambda = v/f$, that is, a constant spacing λ between the crests. The approaching observer sees, in time t, the crests in a distance $(v+v_o)\,t$, or $(v+v_o)t/\lambda$ crests.
Thus the observed frequency f' is (number of crests observed/time) = $(v+v_o)/\lambda = f\,(1+v_o/v)$.

3 marks for clear derivatio-n: fixed λ is key.

The full expression when both source and observer are moving towards each other is

$$f' = f\cdot\frac{\left(1+\frac{v_o}{v}\right)}{1-\frac{v_s}{v}}.$$

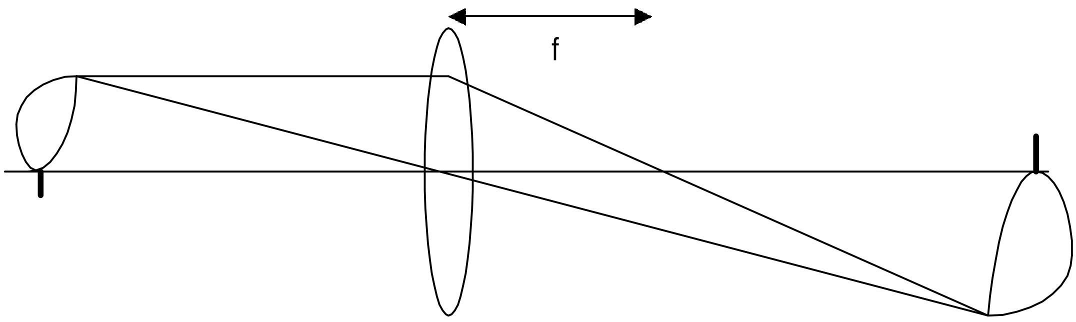
2 marks

When the observer moves at 33 m/s towards a source of 1500 Hz and the speed of sound is 330m/s, the frequency observed will be $1500\cdot\left(1+\frac{33}{330}\right) = 1650$ Hz.

2 marks

5. The initial sentence tells us that the focal length of the lens is 50 mm. The image of a candle flame may be formed as shown below, the image being inverted..

1 mark for focal length



2 marks for diagram - need at least 2 principal rays

1 for stating image is inverted

If the image is formed at a distance *d* from the candle, and the paper is a distance *v* from the lens, the lens formula

$1/v - 1/u = 1/f$

becomes

$1/v + 1/(d-v) = 1/f$.

Multiplying out

$f(d-v) + f\,v = v(d-v)$

or

$v^2 - d\,v + f\,d = 0$.

Solving, with $d := 200 \cdot mm,\ f := 50 \cdot mm$ (note that the square root is zero, so there is only one solution)

$$v = \frac{\left(d + \sqrt{d^2 - 4 \cdot f \cdot d}\right)}{2} = 0.1\ \text{m}\quad .$$

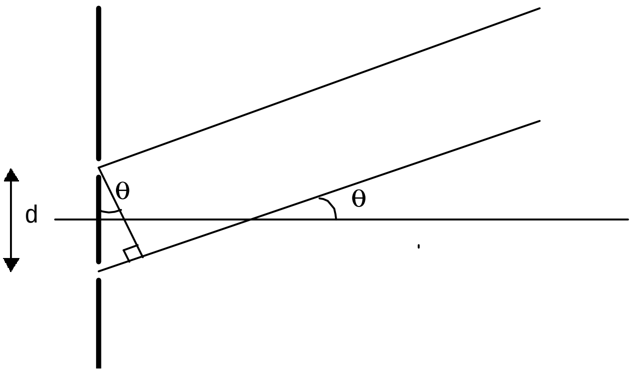
That is, the lens should be placed mid-way between the candle and the paper.

1 mark for lens formula (giving benefit of doubt if sign convention unclear),

2 for rest of calculation.

6. The difference in distance (as shown in the diagram below) is

$d \cdot sin(\theta)$.



3 marks for clear derivation of path difference, of which 1 mark is for setting up the situation,

2 marks for fringe spacing

For constructive interference, forming bright fringes, this path difference must be an integral number of wavelengths. For small angles θ, sin(θ) is approximately equal to θ, and so if bright fringes are formed a distance *y* off-axis at a range *D* we must have

$d\ (y/D) = n\,\lambda$.

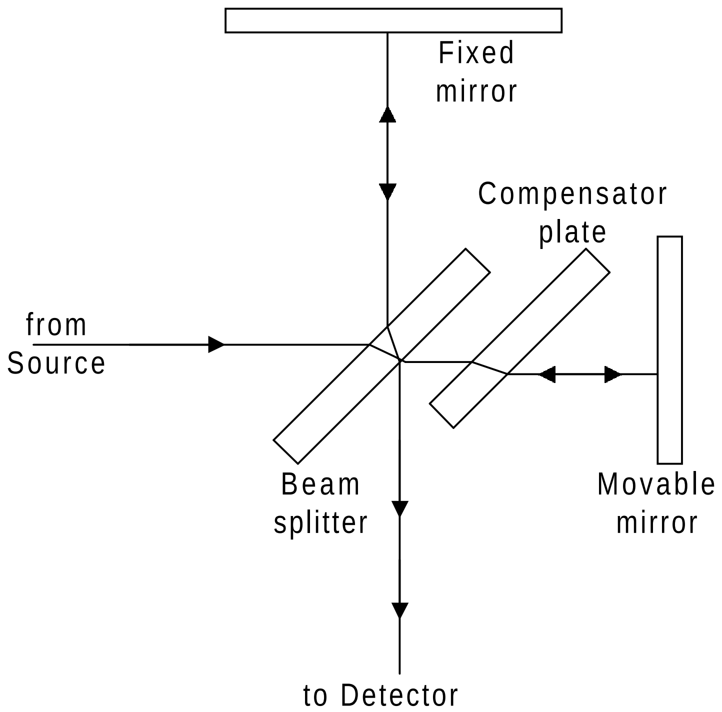
The separation of the fringes, then, is

$\delta y = \lambda\, D/d$. 2 marks

With $d := 0.1 \cdot mm,\ D := 1 \cdot m,\ \delta y := 6 \cdot mm,$ we have for the wavelength of the light

$$\lambda = \frac{d \cdot \delta y}{D} = 6 \cdot 10^{-7}\ \text{m} \quad \text{or 600 nm.}$$

7. The diagram is below.



The mirrors are coated on their front surfaces, the beam splitter is part-silvered on its rear surface.

The compensator plate is the same, except for the silvering, as the beam splitter.

5 marks - MUST mention part-silvering and ability to move one mirror.

A movement d of the movable mirror corresponds to a change 2d in path length difference. Each change in path length of one wavelength causes one fringe to pass. Here 1000 fringes correspond to a change in path length of twice 0.32 mm, so the wavelength is $\frac{2 \cdot 0.32 \cdot mm}{1000} = 6.4 \cdot 10^{-7}$ m or 640 nm.

2 marks

8. Phase velocity v_p is the speed at which points of constant phase in the wave propagate, or the velocity of the carrier wave, or ω/k.

Group velocity v_g is the speed at which a signal, or energy, propagates, or the velocity of the envelope, or $d\omega/dk$.

2 marks
Allow full marks if formulae correct but words not.

$v_g = d\omega/dk = d(k\, v_p)/dk = v_p + k\, dv_p/dk.$

3 marks

For the elastic waves in a thin plate, we have

$v_p = \omega/k = A\, k$

and

$v_g = d\omega/dk = 2\, A\, k$

so the group velocity is double the phase velocity.

2 marks: deduct 1/2 mark if relationship not given explicitly.

$t := 0, 0.1 .. 10$

$v(t) := cos(10 \cdot t) \cdot cos(t)$
$z(t) := cos(t)$

$d := 0.1 \cdot mm$

$\delta y := 6 \cdot mm$

$D := 1 \cdot m$