2. The Mean Square Velocity of molecules in an Ideal Gas

This is the 2nd mathematical derivation of 5 for 1B28 Thermal Physics.

a) For the mean square speed, v_{mean} of the molecules in an ideal gas in 3 dimensions:

We start with an equation of the form

$$\int_0^\infty x^4 e^{-ax^2}\,dx = \frac{3}{8}\sqrt{\frac{\pi}{a^5}}$$

$$\int_0^\infty v^2 P(v)\,dv = \int_0^\infty \left(\frac{m}{2\pi k T}\right)^{3/2} 4\pi\, v^4\, e^{\frac{-mv^2}{2kT}}\,dv$$

$$(4\pi)\left(\frac{m}{2\pi k T}\right)^{3/2}\int_0^\infty v^4\, e^{\frac{-mv^2}{2kT}}\,dv$$

$$4\pi\left(\frac{m}{2\pi k T}\right)^{3/2}\left[\frac{3}{8}\sqrt{\frac{\pi}{\left(\frac{m}{2kT}\right)^5}}\right]$$

$$4\pi\left(\frac{m}{2\pi k T}\right)^{3/2}\left[\frac{3}{8}\sqrt{\frac{2^5\pi\, k^5 T^5}{m^5}}\right]$$

Then check workings by expanding and cancelling to get hopefully:

$$\frac{12}{8}\left(\frac{m}{2\pi k T}\right)\left(\frac{m}{2\pi k T}\right)\frac{2kT}{m}\quad\frac{2\pi k T}{m}$$

$$\frac{12}{8}\quad\frac{2kT}{\pi m} = \frac{24}{8}\quad\frac{kT}{\pi m} = \frac{3kT}{m}$$

$$v = \frac{3kT}{m}$$

b) For the mean speed, v_{mean} in 2 dimensions:

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1. The Maxwell-Boltzmann Speed for the molecules in an Ideal Gas
3. The RMS speed, v_{rms}
4. The mean kinetic energy, E = k.e.
5. Some useful Gaussian integrals