4. The Mean Kinetic Energy of an Ideal Gas in 3 Dimensions

This derivation is the 4th of 5 mathematical derivations for 1B28 Thermal and Kinetic Physics.

a) For the mean k.e. of the gas molecules in 3 dimensions:

$$\overline{k.e.} = \overline{1/2\, m v^2} = m/2 \int_0^\infty f(v)\, v^2 dv$$

$$2\pi m \left(\frac{m}{2\pi kt}\right)^{3/2} \int_0^\infty e^{-\frac{mv^2}{2kT}} v^4\, dv$$

$$\left(\int_0^\infty e^{-ax^2} x^4 dx = 3/8 \sqrt{\frac{\pi}{a}}\right)$$

$$2\pi m \left(\frac{m}{2\pi kT}\right)^{3/2} \left[\frac{3}{8}\sqrt{\frac{\pi}{\left(\frac{m}{2kT}\right)^5}}\right]$$

$$2\pi m \left(\frac{m}{2\pi kT}\right)^{3/2} \left[\frac{3}{8}\sqrt{\frac{2^5 \pi\, k^5 T^5}{m^5}}\right]$$

$$\frac{6\pi m}{8} \left(\frac{m}{2\pi kT}\right) \left(\frac{m}{2\pi kT}\right)^{1/2} \left[\frac{2^2 k^2 T^2}{m^2}\right] \left(\frac{2\pi kT}{m}\right)^{1/2}$$

$$\left(\frac{6\pi m}{8}\right) \left(\frac{2kT}{\pi m}\right) = \frac{12}{8}(kT) = \frac{3}{2}(kT)$$

The factor 1/2 arises to avoid double counting the molecules.

b) For the mean k.e. of the gas molecules in 2 dimensions:

$$\left(\frac{1}{2} m\right)(2\pi) \int_0^\infty v^2 \, P(v)\, dv$$

$$\left[\frac{1}{2} m\right](2\pi) \int_0^\infty \left(\frac{m}{2\pi k T}\right) v^3 \, e^{\frac{-mv^2}{2kT}} \, dv$$

$$(2\pi)\left(\frac{1}{2} m\right)\left(\frac{m}{2\pi k T}\right)\left[\frac{1}{2\left(\frac{m}{2kT}\right)^2}\right]$$

$$\frac{1}{2} m \, (2\pi)\left(\frac{m}{2\pi k T}\right)\left[\frac{4\, k^2 T^2}{2 m^2}\right] = \frac{2}{2}\, k T$$

$$= \overline{k.e.}$$

Note that this is an average over all the molecules in the gas. There will usually be a few slower and faster molecules in the tails of the speed distribution.

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1. The Maxwell-Boltzmann Speed for the molecules in an Ideal Gas
2. The mean velocity v_{mean}
3. The RMS speed, v_{rms}
5. Some useful Gaussian integrals
