1B24

WAVES OPTICS AND ACOUSTICS

REVISION LECTURE

A guide to the basics

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1 Simple Oscillations and their Description

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- 2-D motion — complex plane — complex number representation

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- We represent the wave as a complex exponential, amplitude A,

$$Ae^{i(\omega t - kx)}$$

- Something observable (a displacement, , an electric or magnetic field....) must be real.
- De Moivre's theorem tells us

$$\Re\left[e^{i(\omega t-kx)}\right]=\cos(\omega t-kx)$$

- What if we want a sine wave rather than a cosine?

$$\begin{aligned}\Re\left[e^{i(\omega t-kx-\pi/2)}\right] &= \Re\left[-ie^{i(\omega t-kx)}\right]\\ &= \Re\left[-i(\cos(\omega t-kx)+i\sin(\omega t-kx))\right]\\ &= \Re\left[-i\cos(\omega t-kx)+\sin(\omega t-kx)\right]\\ &= \sin(\omega t-kx)\end{aligned}$$

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- Intensity = energy per area per time

$$\langle I \rangle = \frac{1}{2} Z\omega^2 |A|^2$$

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3.1.1 same frequency – same amplitude

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- beat frequency is the difference between the two original frequencies

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$$\frac{\partial^2 \psi}{\partial t^2} = c^2 \frac{\partial^2 \psi}{\partial x^2},$$

with c being the wave speed.

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representing waves travelling to the right $(ct - x)$ and to the left $(ct + x)$ — note that what determines the direction of travel is the relative sign of the t term and the x term

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c is speed at which peaks and troughs (points of constant phase) move through the medium — phase velocity.

4.1.1 linearity/superposition

We know that as the wave equation is linear, we may superpose solutions and still get a solution which is a solution of the wave equation.

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5.0.2 wave equation

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- Hence nett force is proportional to second derivative of displacement
- Set this equal to mass of section times acceleration

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(1)

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that is, a wave equation with wave speed $c = \sqrt{T/\rho}$. For a wave on a string, the transverse velocity depends on the frequency and the amplitude, and varies with time. The wave velocity is a constant: in a linear wave (the only sort we deal with) it is independent of amplitude, although (dispersion) it may depend on frequency.

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with $y(x,t) = 0$ at $x = 0$, $x = L$ for all t: $x = 0$ gives $a + b = 0$. Two equal and opposite running waves give a standing wave with nodes at the ends of the string

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$$\lambda_n = \frac{2L}{n} \tag{3}$$

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If we had a rod under constant tension, of course, the fractional extension would be constant along the rod, and $\xi = \frac{F}{AY}x$.

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$$\frac{\partial^2 \xi}{\partial t^2} = \frac{Y}{\rho} \frac{\partial^2 \xi}{\partial x^2}. \tag{4}$$

6.2 Elastic waves in a bulk solid

6.2.1 compression waves and shear waves

Remember that different types of wave exist, but that’s about all you need to know.

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$$\frac{\partial^2 \xi}{\partial t^2} = \frac{B_\mathrm{a}}{\rho_0}\frac{\partial^2 \xi}{\partial x^2} \qquad (6)$$

that is, a wave with velocity $v = \sqrt{B_\mathrm{a}/\rho}$.

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that is, a wave with velocity $v = \sqrt{B_\mathrm{a}/\rho}$. We use the adiabatic bulk modulus because the changes which the wave induces are fast compared with the time-scales for heat transfer through the gas.

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The rate at which energy is transferred, the energy flux, is the product of energy density and wave velocity,

$$I = c\langle E \rangle = \frac{1}{2}\omega^2 Z\xi_0^2. \qquad (11)$$

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