where δ_{ij} is the Kronecker delta function. Here $\langle p \rangle$ denotes the mean value of some quantity p. [8]

(c) A spherically-symmetric galaxy is dark-matter dominated and has a gravitational potential

$$\Phi(r) \quad = \quad - \frac{GM_{tot}}{r+a}$$

at a radial distance r from its centre, where a is a positive constant and M_{tot} is the total mass.

A population of stars is distributed within this potential. The stars contribute negligibly to the total density. The system of stars has an isotropic velocity distribution with a velocity dispersion σ that is constant across the galaxy, and has zero net rotation. Assuming that the potential is constant over time, derive an expression for the number density n of stars as a function of radius r and in terms of the number density n_0 of stars at the centre. [6]

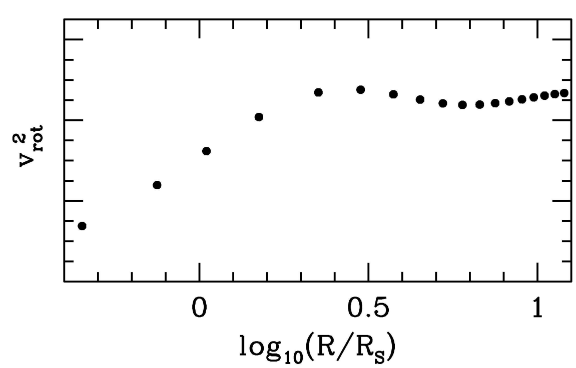
(d) A spherically-symmetric galaxy consists only of stars (it has no dark matter). It has a density distribution for which the internal potential energy is

$$U \quad = \quad - \frac{GM_{tot}^2}{6a} \quad .$$

If the typical velocity of stars in this galaxy is v, calculate the total mass from the virial theorem, in terms of a and v, on the assumption that the system is pressure supported and virialised, and v is constant throughout. [4]

[Total 20 marks for question]

5. (a) The graphs below plot the rotation curve of a spiral galaxy. The first graph shows the circular velocity v_{rot} against R/R_S, where R is the radial distance from the centre and R_S is the exponential scale length of the galaxy's disc observed in visible light. The second plots v_{rot}^2 against $\log_{10}(R/R_S)$.



Identify the velocity maxima in the rotation curve. Which components of the galaxy show a dynamical signature: a bulge, a disc and/or a dark matter halo? Explain your reasoning. [6 marks]

In which part of the electromagnetic spectrum were the observations made? Explain your reasoning. [2]

[This question continues overleaf . . .]