Queen Mary
University of London

M.Sc./M.Sci. EXAMINATION BY COURSE UNITS

ASTM002 / MAS430 The Galaxy

Friday, 30th May, 2008
10:00 a.m. – 1:00 p.m.

Time Allowed: 3 hours

You may attempt as many questions as you wish and all questions carry equal marks. Except for the award of a bare pass, only the best 5 questions answered will be counted.

Calculators ARE permitted in this examination. The unauthorised use of material stored in a pre-programmable memory constitutes an examination offence. Please state on your answer book the name and type of machine used.

YOU ARE NOT PERMITTED TO START READING THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR

Useful information

In this paper π and e represent the conventional mathematical constants.
G represents the gravitational constant with $G = 6.67 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}s^{-2}}$.
c is the velocity of light, with $c = 3.00 \times 10^8\ \mathrm{m\,s^{-1}}$.
1 pc $= 3.09 \times 10^{16}$ m.
The distance of the Sun from the Galactic Centre is $R_0 = 8.0$ kpc.

Poisson's equation states that $\nabla^2\Phi = 4\pi G\rho$ at any point in a gravitational field, where Φ is the gravitational potential, G is the constant of gravitation, and ρ is the density at that point.

The Laplacian of a scalar function Φ in a spherical coordinate system (r, θ, ϕ) is

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2} .$$

The Laplacian of a scalar function Φ in a cylindrical coordinate system (R, ϕ, z) is

$$\nabla^2\Phi = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\Phi}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2} .$$

The Jeans equations in a steady-state (i.e. time-independent), spherically-symmetric galaxy give the following result

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(n\,\langle v_r^2\rangle\right) + \frac{n}{r}\left[\,2\langle v_r^2\rangle - \langle v_\theta^2\rangle - \langle v_\phi^2\rangle\,\right] = -\,n\,\frac{\mathrm{d}\Phi}{\mathrm{d}r} ,$$

in spherical coordinates, where n is the number density of stars at a distance r from the centre, v_r, v_θ and v_ϕ are the components of the velocity in the r, θ and ϕ directions, and $\Phi(r)$ is the gravitational potential.

The virial theorem states that the time-averaged total kinetic energy $\langle T\rangle$ of a system of particles in equilibrium is related to the time-averaged total internal potential energy $\langle U\rangle$ by $2\langle T\rangle + \langle U\rangle = 0$.

In the absence of cosmological effects, the apparent magnitude m of an astronomical object in a photometric band is related to its absolute magnitude M in that band and its distance D from the observer by

$$m - M = 5\log_{10}(D/\mathrm{pc}) - 5 + A ,$$

where A is the extinction in the band expressed in magnitudes.

The fundamental plane for elliptical galaxies is approximately

$$R_S\, I_0^{0.8}\,\sigma_0^{-1.3} = \text{constant} ,$$

where R_S is the scale size of a galaxy, I_0 is its central surface brightness, and σ_0 is the line-of-sight velocity dispersion at its centre.

Oort's constants within the Galaxy are defined as

$$A \equiv \frac{1}{2}\left(\frac{\langle v_\phi\rangle}{R} - \frac{\partial\langle v_\phi\rangle}{\partial R}\right) \qquad \text{and} \qquad B \equiv -\frac{1}{2}\left(\frac{\langle v_\phi\rangle}{R} + \frac{\partial\langle v_\phi\rangle}{\partial R}\right) ,$$

where $\langle v_\phi\rangle$ is the mean tangential velocity in the Galactic disc, and R is the distance from the Galactic Centre, for $R = R_0$.

A standard integral:
$$\int \frac{x^2}{(x^2+a^2)}\,\mathrm{d}x = x - a\tan^{-1}\big(x/a\big) + \text{constant} .$$

[End of the useful information]

1. (a) Briefly describe the observed properties of elliptical galaxies. (You should include reference to their general structure, their stars, gas content, spectra and colours.) [5 marks]

(b) Show that the fundamental plane for elliptical galaxies implies that the luminosity of a galaxy $L \propto \sigma^{2.6}/I_0^{0.6}$, where I_0 is its central surface brightness and σ_0 is the line of sight velocity dispersion at its centre. [3]

(c) The spectrum of a distant galaxy shows a strong continuum with absorption lines, and some emission lines superimposed. What is the morphological type of the galaxy likely to be? Explain your reasoning. [2]

(d) A large stationary gas cloud collapses under its own gravitation. 30 % of the initial potential energy is converted into heat, raising the temperature of the gas. Is this collapse dissipative or dissipationless? [2]

(e) What value does the sum, $A+B$, of the Oort constants have if the rotation curve of the Galaxy is flat near the Sun? [2]
Express the angular velocity of the Galactic disc at the Sun's distance from the centre in terms of A and B. [1]

(f) Explain the term *asymmetric drift.* [2]
How does the asymmetric drift vary with age for stars in the Galactic disc? How does the velocity dispersion of stars depend on age? What effect might giant molecular clouds in the disc of the Galaxy have had on the velocity dispersions of old stars? [3]

[Total 20 marks for question]

2. (a) Show that in a weak encounter between two stars of mass m the change in the velocity v of one star in the reference frame of the other is given by

$$\delta v = \frac{2Gm}{bv} ,$$

[This question continues overleaf . . .]

where G is the constant of gravitation and b is the impact parameter. You may assume that $\int_{-\infty}^{\infty} (k_1 + k_2 s^2)^{-3/2}\, \mathrm{d}s = 2/(k_1\sqrt{k_2})$. [8 marks]

A star moves through a spherical distribution of N stars of overall radius R, with the stars distributed uniformly in space. If the change in the square of the velocity is $\delta v^2 = (\delta v)^2$ in a weak encounter, show that the changes in v^2 caused by all encounters with impact parameters in the range b to $b + \mathrm{d}b$ in a time t is

$$\Delta v^2 = \left(\frac{2Gm}{bv}\right)^2 \left(\frac{3bvtN\mathrm{d}b}{2R^3}\right) .$$ [4]

Hence show that the total change in the square of the velocity in a time t caused by weak encounters with all impact parameters is

$$\Delta v^2(t) = 6 \left(\frac{Gm}{v}\right)^2 \frac{v\,t\,N}{R^3} \ln\left(\frac{b_{max}}{b_{min}}\right) ,$$

where b_{max} and b_{min} are the largest and smallest values of the impact parameter. [4]

Thereby derive the expression for the relaxation time

$$T_{relax} = \frac{1}{6N\ln\left(\frac{b_{max}}{b_{min}}\right)} \frac{(Rv)^3}{(Gm)^2} .$$ [4]

[Total 20 marks for question]

3. (a) Explain briefly what is the distribution function f that used in studying the dynamics of galaxies? [2 marks]

(b) A star moves in an elongated orbit about the Galaxy. As it moves inwards its distance from the Galactic Centre decreases and the star density around it increases. How does f change? How do the velocity dispersions of the stars around it change and why? [3]

(c) The distribution function f in a spherically-symmetric galaxy is related to the mass density $\rho(r)$ at a radial distance r from the centre by

$$\rho(r) = 4\pi\sqrt{2}\,\overline{m} \int_{\Phi(r)}^{0} \sqrt{E_m - \Phi(r)}\ f(E_m)\,\mathrm{d}E_m ,$$

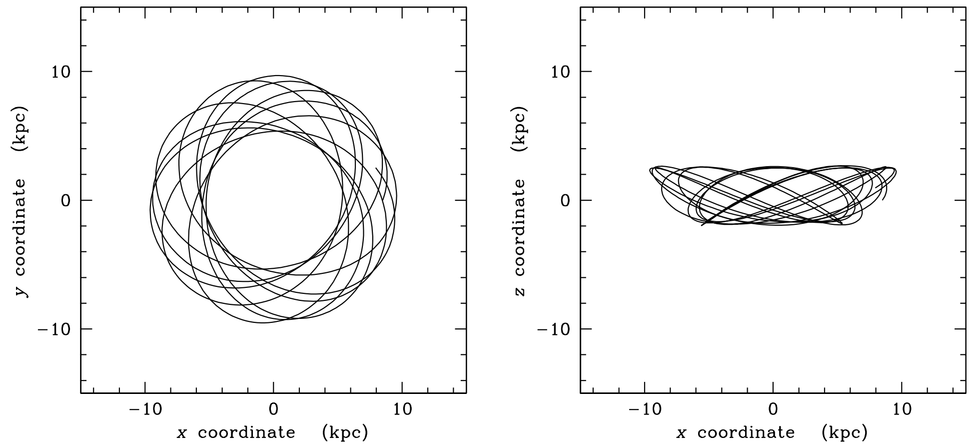
where E_m is the energy per unit mass of a star, $\Phi(r)$ is the gravitational potential at a radius r, and $\overline{m}$ is the mean mass per star. A spherical galaxy is modelled using a potential $\Phi(r)$ and density $\rho(r)$ given by

$$\Phi(r) = -\frac{GM_{tot}}{\sqrt{r^2 + a^2}} \quad \text{and} \quad \rho(r) = \frac{3M_{tot}}{4\pi} \frac{a^2}{(r^2 + a^2)^{5/2}} ,$$

where M_{tot} is the total mass of the galaxy and a is a constant.
Show that a functional form $f(E_m) = b\,(-E_m)^{7/2}$ is a solution for the distribution function for this model where b is a constant. The substitution $E_m = \Phi\cos^2\theta$ and the standard result $\int_0^{\pi/2} \sin^2\theta\ \cos^8\theta\ \mathrm{d}\theta = 7\pi/512$ may prove useful. [9]

[This question continues overleaf . . .]

(d) The diagrams below show the orbit of a star in a gravitational potential, shown projected on to the $x-y$ and the $x-z$ planes.



What do you conclude about the potential: is it (i) spherical, (ii) flattened (oblate), or (iii) triaxial? Justify your answer on the basis of the character of the orbit. [4]

(e) Explain very briefly the concept of violent relaxation. [2]

[Total 20 marks for question]

4. (a) What advantage do the Jeans equations have over the collisionless Boltzmann equation in describing the dynamics and densities of stars in observed galaxies? [2 marks]

(b) The collisionless Boltzmann equation gives

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3} \left(\frac{\mathrm{d}x_i}{\mathrm{d}t} \frac{\partial f}{\partial x_i} + \frac{\mathrm{d}v_i}{\mathrm{d}t} \frac{\partial f}{\partial v_i} \right) = 0,$$

where f is the distribution function, t is time, and x_i and v_i (for $i = 1, 2, 3$) are the components of the position vector $\mathbf{x}$ and velocity vector $\mathbf{v}$ respectively. Derive from this the second Jeans equation,

$$\frac{\partial (n\langle v_i \rangle)}{\partial t} + \sum_{j=1}^{3} \frac{\partial}{\partial x_j} \big(n \langle v_i v_j \rangle \big) = - \frac{\partial \Phi}{\partial x_i} n \ ,$$

where $i = 1, 2$ or 3, $\Phi(\mathbf{x}, t)$ is the gravitational potential, and $n(\mathbf{x}, t)$ is the number density of stars at a point in space. For this you may assume that

$$n\langle v_i \rangle = \int v_i \, f \, \mathrm{d}^3\mathbf{v} \,, \qquad n\langle v_i v_j \rangle = \int v_i \, v_j \, f \, \mathrm{d}^3\mathbf{v} \,,$$

$$\text{and} \qquad \int v_i \frac{\partial \Phi}{\partial x_j} \frac{\partial f}{\partial v_j} \, \mathrm{d}^3\mathbf{v} = - \frac{\partial \Phi}{\partial x_j} \delta_{ij} \, n \ ,$$

[This question continues overleaf . . .]