Queen Mary
University of London

M.Sc. EXAMINATION

ASTM002 The Galaxy

Wednesday, 18 May 2005
18:15 – 21:15

Time Allowed: 3 hours

You should attempt all questions. Marks awarded are shown next to the questions.

Calculators ARE permitted in this examination. The unauthorised use of material stored in a pre-programmable memory constitutes an examination offence. Please state on your answer book the name and type of machine used.

1. (a) Briefly describe the characteristics of the optical spectra of an elliptical galaxy and of an irregular galaxy. [2 marks]

(b) Which chemical elements are responsible for strong lines in the optical spectra of H II regions? Why are forbidden lines observed in the spectra of H II regions but not under normal laboratory conditions? [4]

Observations of an isolated H II region show that the total luminosity through the Hα emission line is 9×10^{47} photons s^{-1}, while it is 5×10^{47} photons s^{-1} in the Hβ line, 4×10^{47} s^{-1} in Hγ, 3×10^{47} s^{-1} in Hδ, and 5×10^{47} s^{-1} in all other Balmer lines. What is the total flux of ultraviolet photons with wavelengths shorter than 912 Å from stars inside the H II region? Why is this wavelength of 912 Å significant? [3]

(c) In which region of the electromagnetic spectrum is the CO molecule most easily observed? What type of energy level transitions are responsible for this emission? Why are observations of CO emission used to trace the distribution of cold molecular gas in the Galaxy, rather than observations of H_2 molecules directly? [4]

(d) A star lying near to the Galactic plane is observed to have a visual magnitude of $V = 15.30$ and a blue magnitude of $B = 16.20$. Spectroscopy shows the star to be of a type that has an intrinsic colour $(B-V)_0 = 0.60$ mag. What is the colour excess of the star? Estimate the extinction in the V band towards the star. [3]

(e) The optical depth caused by dust extinction when light of wavelength λ travels a distance $\mathrm{d}l$ through the interstellar medium can be taken to be $\mathrm{d}\tau_\lambda = \kappa_\lambda \, \rho \, \mathrm{d}l$, where ρ is the density of dust in space and κ_λ can be taken to be a constant. Assuming that the density of dust varies with distance z above the Galactic plane as $\rho(z) = \rho_0 \, \mathrm{e}^{-|z|/h}$, where ρ_0 and h are constants, derive an expression for the total optical depth τ_λ caused by dust extinction along a line of sight through the Galaxy to distant sources as a function of the galactic latitude b. If the extinction in magnitudes, A_λ, is related to the optical depth τ_λ by $A_\lambda = 1.086\,\tau_\lambda$, show that the extinction is given by $A_\lambda = 1.086\,\kappa_\lambda\,\rho_0\,h\,\mathrm{cosec}|b|$. [3]

(f) Under the Simple Model of galactic chemical evolution, the change δZ in the metallicity Z of interstellar gas in a closed volume of space in a short interval of time is related to the change δM_{metals} in the mass M_{metals} of heavy elements in the gas by

$$\delta Z = \frac{\delta M_{\rm metals}}{M_{\rm gas}} - Z\,\frac{\delta M_{\rm gas}}{M_{\rm gas}}$$

where M_{gas} is the mass of gas in the volume. The change in the mass of heavy elements is related to the change δM_{stars} in the mass M_{stars} of stars in the volume by

$$\frac{\delta M_{\rm metals}}{M_{\rm gas}} = -\,Z\,\frac{\delta M_{\rm stars}}{M_{\rm gas}} + p\,\frac{\delta M_{\rm stars}}{M_{\rm gas}}\,,$$

where p is the yield of heavy elements.

Derive from these expressions the relation $Z = -\,p\,\ln\mu$, where $\mu \equiv M_{gas}(t)/M_{total}$ is the fraction of the total mass in the volume in the form of gas.

How does the prediction of the variation of metallicity with gas fraction from this equation compare with observations in irregular galaxies? [6]

2. (a) Explain the difference between pressure support and rotational support for a galaxy. [2 marks]

(b) The dark matter of our Galaxy is sometimes modelled using a spherically-symmetric density distribution

$$\rho(r) = \frac{\rho_0}{1 + r^2/a^2}\,,$$

where $\rho(r)$ is the mass density at a distance r from the centre, and ρ_0 and a are constants. Show that the mass interior to a radius r is

$$M(r) = 4\pi\rho_0\,a^2\,(\,r - a\tan^{-1}(r/a)\,)\,.$$

The substitution $r = a\tan\theta$ and the standard result $\int \tan^2 x\,\mathrm{d}x = \tan x - x +$ constant may prove useful in performing the integration. [6]

(c) Derive an expression for the circular velocity v_{circ} for this mass distribution. What is the dependence of v_{circ} on radial distance r at large distances ($r \gg a$)? How does this compare with the observed rotation curve of the Galaxy? [5]

(d) What is the total mass out to all radii implied by this density profile? How must the real density profile of the Galaxy behave at large radii compared with this model profile? [3]

(e) A galaxy is modelled using a spherically-symmetric gravitational potential of the form

$$\Phi(r) = -\,\frac{GM_{tot}}{a}\,\ln\left(\frac{r+a}{r}\right)\,,$$

where r is the radial distance from the centre of the galaxy, a is a constant, M_{tot} is the total mass of the galaxy and G is the constant of gravitation. Using Poisson's equation $\nabla^2\Phi = 4\pi G\rho$, derive an expression for the mass density ρ as a function of distance r implied by this potential. You may assume that in a spherical polar coordinate system (r, θ, ϕ), the Laplacian of a function A is

$$\nabla^2 A \equiv \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial A}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial A}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 A}{\partial\phi^2}\,.$$

What is the behaviour of this potential $\Phi(r)$ as $r \to \infty$ and as $r \to 0$? Comment on whether this behaviour is physically realistic. [6]

(f) Observations of radial velocities of the dwarf companions to our Galaxy can be used to estimate the total mass of the Galaxy out to radial distances > 100 kpc. In one model, these companion galaxies are assumed to be falling towards the Galaxy in radial paths, while in another model they are assumed to be following orbits that have similar velocity components perpendicular to our line of sight as they have towards us. How do these assumptions affect the estimated mass of the Galaxy? Which estimate is the larger? [3]

3. (a) What is the distribution function f that is used in studying the dynamics of galaxies? [2 marks]

(b) The gravitational potential of our Galaxy can be regarded as being axisymmetric to a good approximation. In an axisymmetric potential, which of the following act as integrals of motion in solving for the stellar distribution function f: (i) the total energy of a star; (ii) the total angular momentum; (iii) the component of the angular momentum in the direction parallel to the axis of symmetry? [3]

(c) The distribution function f in a spherically-symmetric galaxy is related to the mass density $\rho(r)$ at a radial distance r from the centre by

$$\rho(r) = 4\pi\sqrt{2}\,\overline{m}\int_{\Phi(r)}^{0}\sqrt{E_m - \Phi(r)}\;f(E_m)\,\mathrm{d}E_m\,,$$

where E_m is the energy per unit mass of a star, $\Phi(r)$ is the gravitational potential at a radius r, and $\overline{m}$ is the mean mass per star. A spherical galaxy is modelled using a potential $\Phi(r)$ and density $\rho(r)$ given by

$$\Phi(r) = -\frac{GM_{tot}}{\sqrt{r^2 + a^2}} \quad \text{and} \quad \rho(r) = \frac{3M_{tot}}{4\pi}\,\frac{a^2}{(r^2 + a^2)^{5/2}}\,,$$

where M_{tot} is the total mass of the galaxy, G is the constant of gravitation, and a is a constant.

Show that a functional form $f(E_m) = b\,(-E_m)^{7/2}$ is a solution for the distribution function for this model. What is the required value of the constant b? The substitution $E_m = \Phi\cos^2\theta$ and the standard result

$$\int_0^{\frac{\pi}{2}} \sin^2\theta\,\cos^8\theta\,\mathrm{d}\theta = \frac{7\pi}{512}$$

 [This question continues overleaf . . .]

may prove useful. [7]

(d) What advantage do the Jeans equations have over the collisionless Boltzmann equation in describing the dynamics and densities of stars in observed galaxies? [2]

(e) The collisionless Boltzmann equation gives

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3} \left(\frac{\mathrm{d}x_i}{\mathrm{d}t} \frac{\partial f}{\partial x_i} + \frac{\mathrm{d}v_i}{\mathrm{d}t} \frac{\partial f}{\partial v_i} \right) = 0 \,,$$

where f is the distribution function, t is time, x_i for $i = 1, 2, 3$ are the components of the position vector $\mathbf{x}$, and v_i for $i = 1, 2, 3$ are the components of the velocity vector $\mathbf{v}$.
Derive from this the first of the Jeans equations,

$$\frac{\partial n}{\partial t} + \sum_{i=1}^{3} \frac{\partial n \langle v_i \rangle}{\partial x_i} = 0 \,,$$

where n is the number density of stars and $\langle v_i \rangle$ is the mean value of the v_i velocity component at a point. [5]

(f) The second Jeans equation in a spherically-symmetric potential gives

$$\frac{\partial}{\partial r}\Big(n \, \langle v_r^2 \rangle \Big) + \frac{n}{r} \Big[\, 2\langle v_r^2 \rangle - \langle v_\theta^2 \rangle - \langle v_\phi^2 \rangle \Big] = - \, n \, \frac{\partial \Phi}{\partial r} \,,$$

for a spherical polar coordinate system (r, θ, ϕ), where $n(r)$ is the number density of stars in space, $\Phi(r)$ is the gravitational potential, and v_r, v_θ and v_z are the components of the velocity in the r, θ and ϕ directions at a point.
A galaxy has a gravitational potential at a distance r from its centre that is given by

$$\Phi(r) = - \frac{G M_{tot}}{\sqrt{r^2 + a^2}} \,,$$

where M_{tot} is the total mass, a is a constant and G is the constant of gravitation. Assuming that the velocity dispersion σ of a population of stars is isotropic and constant over the whole galaxy, and that there is no net rotation, show that the number density of these stars in this potential is

$$n(r) = n_0 \, \exp \left[\frac{G M_{tot}}{a \, \sigma^2} \left(\frac{1}{\sqrt{1 + r^2/a^2}} - 1 \right) \right] \,,$$

where $n_0 = n(0)$. [6]

[Next question overleaf.]

4. (a) A star is observed to have [Fe/H] $= -2$. What is the chemical abundance of iron relative to hydrogen in the star compared to the Sun? [1 mark]
A star has [Fe/H] $= -1.54$. Which component of the Galaxy would you expect it to belong to? What would you expect its age to be? [2]
Metal-poor stars are often found to have significantly higher oxygen abundances relative to iron than the Sun. How might this be explained in terms of the timescales for enrichment of the interstellar medium by supernovae? [3]

(b) Explain the term *asymmetric drift.* [2]
How does the asymmetric drift vary with age for stars in the Galactic disc? How does the velocity dispersion of stars depend on age? What effect might giant molecular clouds in the disc of the Galaxy have had on the velocity dispersions of old stars? [3]

(c) It has been suggested that the dark matter halo of the Galaxy might be in the form of low mass stars or brown dwarfs (with masses $M \sim 0.05 M_\odot$, where $M_\odot = 2.0 \times 10^{30}$ kg is the mass of the Sun). Another opinion is that it might consist of elementary particles with masses $\sim 10^{-24}$ kg. Using order of magnitude estimates, show that the gravitational lensing effect of a brown dwarf in the halo would be large enough to be detected by an optical microlensing survey of distant stars in the Magellanic Clouds, but such a survey would be insensitive to lensing by a subatomic particle at the same distance. You may use the result that the angle corresponding to the Einstein radius of a lensing object of mass M is

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{\rm LS}}{D_{\rm L}D_{\rm S}}} \quad \text{radians} ,$$

where D_L is the distance between the observer and lens, D_S is the distance between the observer and source, D_{LS} is the distance between the lens and source, $G = 6.7 \times 10^{-11}\ {\rm N\,m^2\,kg^{-2}}$ is the constant of gravitation, and $c = 3.0 \times 10^8\ {\rm m\,s^{-1}}$ is the speed of light. The radius of the Sun is $R_\odot = 7.0 \times 10^8$ m and one parsec $= 3.1 \times 10^{16}$ m. [6]

(d) Over the past several years microlensing surveys have tested the hypothesis that the dark matter halo is composed of low mass stars or brown dwarfs. What have these indicated? [2]

(e) The monolithic collapse scenario was once the preferred model for the formation of the Galaxy. Briefly explain how in this model, through the collapse of a single protogalactic cloud with some net angular momentum, the main components of the Galaxy could have been formed. Include an explanation of how the main kinematic, age and metallicity properties of these components arise in the model. [5]

(f) An alternative model for the formation of galaxies proposes that clumps of dark matter containing embedded baryonic matter merged to form the galaxies we observe today. Which of these two models is supported by modern computer simulations? [1]

[End of examination paper.]