Queen Mary
University of London

M.Sc. EXAMINATION

ASTM002 The Galaxy

Wednesday, 19 May 2004
10:00 – 13:00

Time Allowed: 3 hours
You should attempt all questions. Marks awarded are shown next to the questions.
Calculators are NOT permitted in this examination.

1. (a) Briefly state how the observed colours of galaxies depend on their morphological types. [2 marks]

(b) How does the gas content vary with galaxy type? [2]

(c) Explain the difference between collisional and collisionless encounters. [3]

(d) Is gas collisional or collisionless in an astronomical context? [2]

(e) Show that in a weak encounter between two stars of mass m the change in the velocity v is given by

$$\delta v = \frac{2Gm}{bv} \ ,$$

where G is the constant of gravitation and b is the impact parameter. You may assume that $\int_{-\infty}^{\infty} (1 + s^2)^{-3/2} \, \mathrm{d}s = 2$. [8]

(f) The relaxation time T_{relax} due to stellar encounters in a system is related to the crossing time T_{cross} by

$$\frac{T_{relax}}{T_{cross}} \simeq \frac{N}{6 \ln N} \simeq \frac{N}{14 \log N} \ ,$$

where N is the number of stars. A globular cluster with 10^5 stars has a crossing time of $T_{cross} = 5 \times 10^5$ years, while a galaxy with 10^{11} stars has $T_{cross} = 3 \times 10^7$ years. Are two-body encounters significant in these stellar systems? Explain why. [6]

(g) What significance does this have for modelling the dynamics of galaxies? [2]

2. (a) The continuity equation in stellar dynamics states that

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3} \left(\frac{\partial}{\partial x_i}\Big(f \frac{\mathrm{d}x_i}{\mathrm{d}t}\Big) + \frac{\partial}{\partial v_i}\Big(f \frac{\mathrm{d}v_i}{\mathrm{d}t}\Big) \right) = 0 \ ,$$

where f is the distribution function in the phase space defined by space coordinates x_i and velocity coordinates v_i ($i = 1$ to 3), and t is time. Derive from this the collisionless Boltzmann equation

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3} \left(\frac{\mathrm{d}x_i}{\mathrm{d}t}\frac{\partial f}{\partial x_i} + \frac{\mathrm{d}v_i}{\mathrm{d}t}\frac{\partial f}{\partial v_i} \right) = 0 \ ,$$

showing your working. [6]

(b) A star moves in an elongated orbit about the Galaxy. As its distance from the Galactic Centre increases, the star density around it decreases. How does f change? How do the velocity dispersions of the stars star change and why? [4]

(c) The Jeans equations in a cylindrical coordinate system (R, θ, z) centred on the Galaxy, with $z = 0$ in the plane, give

$$\frac{\partial(n\langle v_z\rangle)}{\partial t} + \frac{\partial(n\langle v_R v_z\rangle)}{\partial R} + \frac{\partial(n\langle v_z^2\rangle)}{\partial z} + \frac{n\langle v_R v_z\rangle}{R} = -\,n\,\frac{\partial \Phi}{\partial z} \ ,$$

where n is the star number density, v_R and v_z are the velocity components in the R and z directions, $\Phi(R, z)$ is the Galactic gravitational potential and t is time. Given that the second and fourth terms are found to be negligible, derive an expression for the surface mass density $\Sigma(z, R_0)$ within a distance z of the midplane of the Galactic disc at the solar radius R_0 using the Poisson equation $\nabla^2\Phi = 4\pi G\rho$. [8]

(d) Explain how this enables the surface mass density of the Galactic disc to be found from observations of stars. [5]

(e) Does any significant quantity of dark matter exist within the Galactic disc? [2]

3. (a) List any four assumptions behind the Simple Model of galactic chemical enrichment. [4]

(b) The change in heavy element mass fraction δZ when the mass of gas M_{gas} in a volume of space changes by δM_{gas} is given under the Simple Model by

$$\delta Z = -\,p\,\frac{\delta M_{gas}}{M_{gas}} \ ,$$

where p is the yield of heavy elements. Show that the mass $M_{stars}(t)$ of stars at time t is related to the heavy element fraction $Z(t)$ and the initial gas mass $M_{gas}(0)$ by

$$M_{stars}(t) = M_{gas}(0)\,(1 - \mathrm{e}^{-Z/p}) \ .$$

[8]

(c) Hence explain why the metallicity distribution of a long-lived sample of stars can be described by

$$\frac{N(Z)}{N_1} = \frac{1 - \mathrm{e}^{-Z/p}}{1 - \mathrm{e}^{-Z_1/p}} \quad ,$$

where $N(Z)$ is the number of stars with a metallicity less than Z, and Z_1 and N_1 are the current values of Z and $N(Z)$. [5]

(d) How do the predictions of the Simple Model compare with observations? [4]

(e) How would the comparison be changed if the gas in the model had already been seeded with heavy elements before the Galaxy formed? [2]

(f) What effect would the loss of enriched gas from the interstellar medium have on the metallicity distribution? [2]

4. (a) Rotation curves of spiral galaxies can be measured from optical spectra and from 21cm radio observations. Why is it often difficult to determine the presence of dark matter unambiguously from optical spectra? [2]

(b) Why are 21cm observations preferred for studying galaxy rotation curves? [2]

(c) What is the origin of the 21cm radio emission from the interstellar medium of the Galaxy? [2]

(d) Why is this line impossible to observe directly in the laboratory. [1]

(e) What is meant by the *optical depth* in gravitational lensing? [2]

(f) The angular size corresponding to the Einstein radius of a gravitational lens is

$$\theta_E = \sqrt{\frac{4GM_L}{c^2}\frac{D_{LS}}{D_L D_S}} \quad ,$$

where M_L is the mass of the lens, and D_S, D_L and D_{LS} are the distances from the observer to the light source, from the observer to the lensing object, and between the lens and source respectively. G is the universal gravitational constant and c is the velocity of light. Show that the optical depth through a distribution of microlenses of mass M_L along a path length to a source is given by

$$\tau = \frac{4\pi G}{c^2 D_S} \int_0^{D_S} D_L \, D_{LS} \, \rho(D_L) \, \mathrm{d}D_L \quad ,$$

where ρ is the mean mass density of lenses in a volume of space. [7]

(g) Assuming that the dark matter halo of the Galaxy can be represented by an isothermal sphere of compact objects with a distribution $\rho(r) = k/2\pi G r^2$ where r is the radial distance from the Galactic centre and k is a constant, show that the optical depth for microlensing towards a globular cluster at the north galactic pole is

$$\tau = \frac{k}{2c^2} \left(2\ln 2 + \pi - 4 \right) \quad ,$$

 [This question continues overleaf ...]

if the distance to the cluster is the same as the distance R_0 of the Sun from the Galactic centre. You may assume that

$$\int \frac{x}{a^2 + x^2}\, \mathrm{d}x \;=\; \frac{1}{2} \ln \left(a^2 + x^2 \right) \;+\; \text{constant}$$

and

$$\int \frac{1}{a^2 + x^2}\, \mathrm{d}x \;=\; \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) \;+\; \text{constant}$$

[7]

(h) Why in practice are star fields in the Large Magellanic Cloud and the Galactic bulge used for microlensing studies, rather than stars in random fields? [2]

 [End of examination paper.]